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Numbers in the Sky(viewing Sculpture)

Branko Ćurgus
Western Washington University, branko.curgus@wwu.edu

"NUMBERS IN THE SKY(VIEWING SCULPTURE)"

DR. BRANKO ĆURGUS
PROFESSOR, MATHEMATICS DEPARTMENT

The following is my personal exploration of Noguchi's sculpture inspired by what I do most of the time: math, or playing with numbers. His sculpture is very geometric; it lends itself to mathematical explorations, and I decided to look for a mathematical message in it.

I think *Skyviewing Sculpture* is really beautiful, and I have also encountered a lot of beautiful things in math, so the natural thing was to look for a connection: the beautiful sculpture giving rise to beautiful math. This is a short report of what I found. There is much more to be discovered, and these are the first findings. Let me give you some background.

I started doing this in '95 in calculus classes that I taught at that time. As an exercise, I asked students to calculate the volume of the sculpture. I hope each one of you has had some experience of finding areas of geometric objects. When you want to carpet your room you have to figure out how much carpet you need. You measure the sides of the room; you get the numbers A and B, and you multiply numbers A and B to get the area of your room. If you are shopping for an air purifying system, they will ask you about the volume of your room. To find out the volume of your room, you'll have to multiply this area of the base with the height of the room; that will give you the volume. That is a simple volume to calculate. Obviously it is much more challenging to calculate the volume of the sculpture. Therefore, it is an assignment that requires creative explorations on the part of the students.

This assignment caught the attention of the *Klipsun*, and a student journalist called me in February 2002. She read me the sentence that was to be published in the *Klipsun* about what I was doing in my classes. The sentence read: "Within the metal lies a math equation, which math professor Branko Ćurgus has had his students solve in the past." I was really pleased. *(Laughter) W*ho knows? Maybe I'll be famous or something. *(Laughter)* So I was looking forward to seeing the *Klipsun*. When it came out I read this sentence and was very pleased. Then, you know, the next sentence read: "This is just one example of how those who aren't interested in art can relate to the sculptures." *(Laughter)* How is this possible? Why is it so that relating math with the sculpture implies

Professor Branko Ćurgus deep in thought.

that I was not interested in art? I thought that I should write to the *Klipsun*, but decided to wait for this opportunity to make my statement. I am interested in art. This is, in the same way as Daniel wrote his poem, my way of celebrating Noguchi's beautiful piece of art.

This leads me to ask: Why is it so that math and numbers have a bad public image? I think professional mathematicians enjoy what they do, but they haven't been very successful in getting across the beauty of their subject to the general public. My suggestion is that the American Mathematical Society should hire Saatchi & Saatchi, the advertising agency, to improve the public image of our field.

Now I will briefly go into some heavy-duty math. Before I do that let me tell you that numbers, same as words, can be exciting, intriguing, and mysterious. I will list some of the most remarkable numbers in mathematics that I call "The Hall of Fame of Numbers." I have ordered them in the progression of how one learns all these numbers in our personal mathematical experiences. One (1) is a number that one discovers first. A baby gets its own first pacifier, this is One (1) pacifier. Very soon it will discover Zero (0), as its pacifier is taken away. And then, it may take a little bit longer, but one learns about Pi (π). Then there are more complicated numbers: $\boldsymbol{e}$ and $\boldsymbol{i}$. One learns about $\boldsymbol{e}$ in business algebra or calculus. One has to take even more math to learn about $\boldsymbol{i}$. These are remarkable concepts that we teach in math. One of the most fascinating equations in mathematics is the Euler's equation, which magically brings all these numbers into ONE equation:

$$\boldsymbol{e}^{i\pi} + 1 = 0$$

It takes probably three academic quarters of calculus to understand this equation. Believe me, it's fascinating and worth every minute of the time invested. I have been thinking about this equation for many years, and I have even made attempts to make a popular presentation about it. The fascination of this equation is that it includes all numbers from "The Hall of Fame of Numbers": 1, 0, π, $\boldsymbol{e}$, and $\boldsymbol{i}$. *(Laughter)* This beautiful equation inspired me to look for a beautiful equation in Noguchi's sculpture.

What equation is hidden in *Skyviewing Sculpture?* Let's look for it in the sculpture's volume. I'm not going to tell you what the volume is in terms of cubic feet. In math we are not big on units; we like to be free and choose our own units. So, "One" (1) is just an abstract concept that is most commonly represented as a length.

Each one of us, me included, has had difficulties in dealing with numbers; one reason being that there are simply too many of them. But I found that one way to overcome those difficulties is to visualize them. To give each number a different flavor I also like to color each with a different color. So I colored One (1) sea green. You can also visualize One (1) as an area, so this is "area One (1)." [Slide 1] So that is what I can say about One (1).

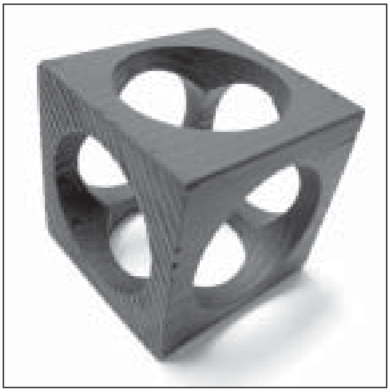
Now I will challenge you with a mathematical problem because I believe that you are quite up to it. Which number is represented by the area surrounded by the thick black line? [Slide 2] *(Audience: four).* Yes, Four (4). So now, we see the number Four (4). Next I will introduce you to a number that mystified ancient Greeks. Which number is represented by the sky blue area? [Slide 2] Yes, the number Two (2) is sky blue. Which number is navy blue? [Slide 3] The navy blue number is the edge of the square, which is sky blue. And so what is the navy blue length? *(Audience: square root of two.)* Yes, it is the mysterious number. There are too many math students here, I apologize. *(Laughter)* People have been mystified for centuries by the number square root of Two. This picture [Slide 3] shows that it is very closely related to "One" (1). When you draw a square with the side One the next thing you discover is the square root two as the length of the diagonal. Now we see sea green One (1) and navy blue square root of Two (2).

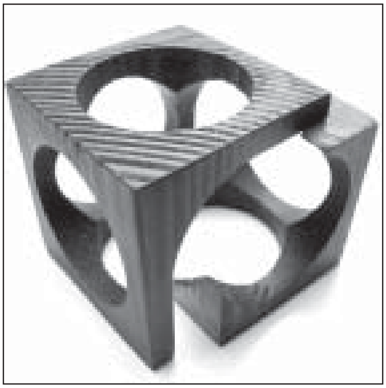
Next is, of course, yellow π. One of the most powerful representations of π is the area of a disk with radius One (1). [Slide 4] You see that sea green One is the radius of the yellow area π. In this way you see π.

The next number in the Hall of Fame of Numbers is ***e***. I'm not going to talk about ***e***, since ***e*** is a tough concept. To get this concept across I created an ***e***-shirt, that is a tee shirt that celebrates the power of ***e***. On the other hand, as we have seen π is just yellow, that's not hard.

Now compare white, yellow, and sky blue areas. [Slides 5 & 6] If these were rooms, which room would you prefer? *(Audience: white).* White is the largest area! White is larger than yellow. How does sky blue compare to this? It is even smaller. Do you agree? White is larger than yellow, and yellow is larger than sky blue. In terms of numbers: Two (2) is less than π, and π is less than Four (4). We have just proved that π is between Two (2) and Four (4). The point here is that we have *proved* it. You all know that π is approximately 3.14. We could prove this fact in a similar way, but that would take more time.

Well, how is this related to the sculpture? Noguchi decided that the yellow π is too tightly squeezed into the white square. [Slide 5] Therefore he gave it more space by extending the white square by the red number, which I will call "Noguchi number" (***v***). *(Laughter)* [Slide 7] This number still represents a mystery to me. In my understanding, it somehow brings harmony to the sculpture: the balance between the sculpture and its empty space is based on this number. This number is the key to the sculpture, but I don't have much to say about it at this time. Maybe next time.





(Top) Picture 1 - *A strong resemblance of a cube is retained.*

(Bottom) Picture 2 - *A corner is removed form the cube.*

Now [Slide 8] we see the connection among previously discussed geometric objects and the sculpture: with the yellow area removed we get one of the sides of the sculpture. The first step to calculating the volume of the sculpture is to calculate the area of the black square. To calculate this area is a middle school problem.

From this square I will go to a simple volume. [Slide 9] This is just a box with a square base. This box is somewhere in the sculpture and the thickness of the box is the Noguchi number. To calculate its volume is also a middle school problem. To get a step closer to the sculpture we want to remove the cylindrical shape from this black box. I will remove the yellow cylinder. I will just lift it ... [Slide 10] *(Laughter)* I will remove the cylinder; it's removed. The volume of the cylinder is "Noguchi number" multiplied by π. The remaining geometric object is clearly a piece of the sculpture. See the next slide [Slide11] for its volume.

Since now we are dealing with a three-dimensional object, I think that it will be more fun to have a model, although it can also be done on the computer. I asked the wonderful people in the Scientific Technical Services to make me a model of the sculpture. In fact, they made several models.

Next I will demonstrate how the sculpture can be created from a cube. First drill three cylindrical holes through each pair of parallel sides of the cube. [Picture 1]

This shape still preserves a strong resemblance of the cube, but you can also see the sculpture emerging. To get to the sculpture, we choose one of the corners and remove the entire legs attached to that corner. [Picture 2 & 3] In this way we get a beautiful model of the sculpture.

Remember that we are interested in the volume of the sculpture. To calculate the volume of the sculpture we identify and remove parts of the sculpture for which volumes are relatively easy to calculate similar to what was shown in Slide 11. After six of such pieces have been removed we are left with four congruent pieces for which the volume is hard to calculate. I see this shape as the mathematical heart of the sculpture. This shape cannot be simplified any further. It is basically the only complicated volume in the sculpture. It is a sculpture in its own right; it's really a beautiful, beautiful piece. See it in its full glory in Picture 4.

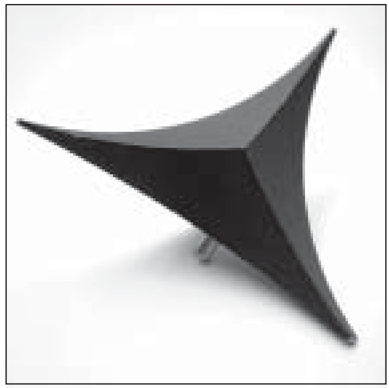
It is in fact a challenging calculus problem to calculate its volume. It can be done in several ways, but it always involves integrals, a concept that is taught in the second quarter calculus class.

In mathematics we are often dealing with beautiful things, but sometimes in order to get to the core information we have to do a tedious calculation which might spoil the fun. In the last decades there has been amazing advancement in calculational tools that remove this tediousness. Just simply hit the enter key. *(Laughter)* To find out the volume that we seek we need to calculate the following integral.

$$\left(1-\frac{1}{\sqrt{2}}\right)^3+3\int_0^{\frac{1}{\sqrt{2}}}\left(1-\sqrt{1-x^2}\right)^2dx$$

Let the mathematical software called *Mathematica* do the job for us. The number is

$$1+\sqrt{2}-\frac{3}{4}\times\pi$$



(Top) Picture 3 - *The corner is completely removed.*

(Bottom) Picture 4 - *The mathematical heart of the sculputure.*

Wow! This is the number, this is the volume of the beautiful piece. It is One plus Square Root Two minus Three-quarters times Pi. All numbers that we already encountered! Not just that they are all there; there is a sort of rhythm to these numbers: One, Two, Three, Four, and instead of Five it is π. All of these numbers are in the sculpture. "One," as I said before, is the unit that we chose: the radius of the circle. We also saw that whenever we have a circle the Square Root Two is present. Sarah mentioned Three several times. Three is in the sculpture. Do you agree, Sarah?

Professor Ćurgus with models of Skyviewing Sculpture.

There are three legs. But Four is powerfully present, too. There are four corners in the air; there are four complicated pieces whose volume is hard to calculate. Pi, as the yellow volume, is present in the sculpture as an open space.

So all these numbers announce themselves powerfully when we approach the sculpture. Amazingly, combined as in the formula above with all basic algebraic operations (+, –, x, ÷, √) they give the only complicated volume in the sculpture. The number

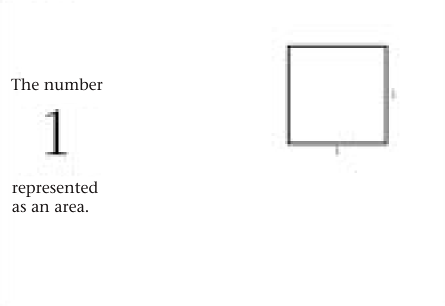
$$1 + \sqrt{2} - \frac{3}{4} \times \pi$$

is the mathematical spirit of the sculpture. And, you know what? [Slide 12] I know that I'm right; you know why? Because here, Isamu is celebrating the special shape in the corner of the sculpture and its volume that we calculated. *(Applause and laughter)*

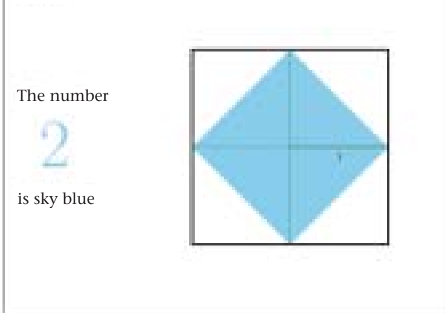
[Professor Ćurgus brought kits for each member of the audience to make a model of the sculpture from Styrofoam cubes and few simple tools designed by himself and provided by the Scientific Technical Services.]

SLIDES

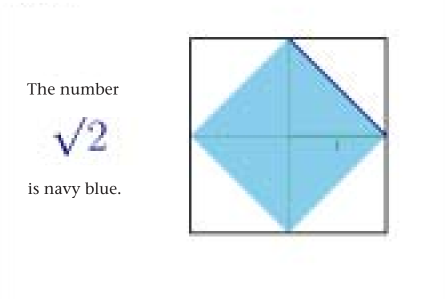
Slide 1



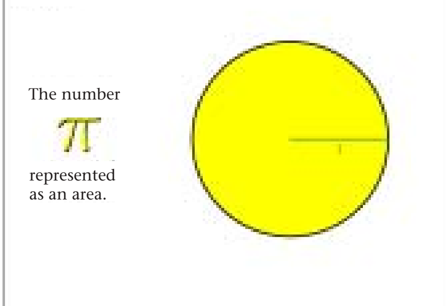
Slide 2



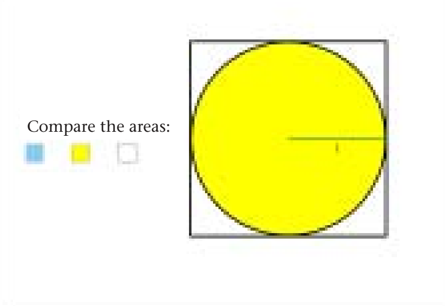
Slide 3



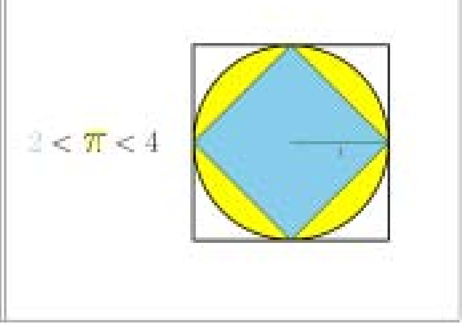
Slide 4



Slide 5

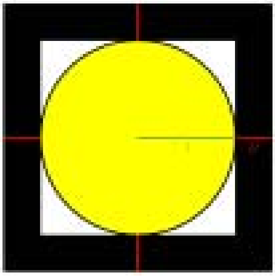


Slide 6

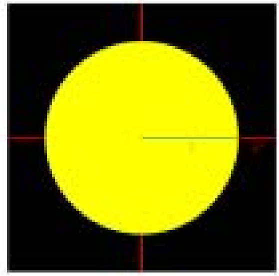


Slide 7

$\nu \approx 0.357196$



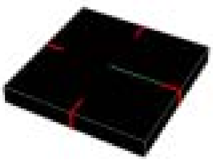
Slide 8



Slide 9

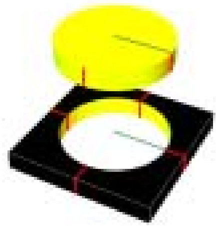
The volume of
the black box is

$4\,\nu\,(1+\nu)^2$



Slide 10

Removing the
yellow cylinder.



Slide 11

The remaining
volume is

$4\,\nu\,(1+\nu)^2 - \nu\,\pi$

Slide 12

$1 + \sqrt{2} - \frac{1}{2}\,\pi$