Real World Math: Engaging Students through Global Issues, Teacher Guide

Facing the Future, Western Washington University

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Real World Math
Engaging Students through Global Issues

Teacher’s Guide

Facing the Future
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Facing the Future is a nonprofit organization dedicated to educating and motivating today’s students to be responsible stewards of tomorrow’s world. We develop and deliver standards-based hands-on lessons, student textbooks, curriculum units, and professional development opportunities for educators that promote critical thinking on global issues, sustainability and positive solutions. Facing the Future curriculum is in use in all 50 U.S. states and over 85 countries by teachers and students in grades K-12, in undergraduate and graduate classes, and across multiple subject areas.

For more information, visit www.facingthefuture.org.
We dedicate this book to Anne Fox, whose tireless dedication to education for sustainability inspires us.
Contributions and Thanks

This book was developed in alignment with national and state mathematics education standards, and with the input of a number of mathematics experts and educators. The standards addressed are indicated for each lesson, and all references are listed at the end of the text.

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Introduction for Educators

When introducing new mathematics concepts to students, you may have heard them ask, “When will I ever use this?” and “How does this affect me?” Real World Math: Engaging Students through Global Issues promotes student engagement by providing real data on global issues with a focus on practical solutions.

Foundational algebra and geometry skills are taught here within the context of global issues and sustainability. Sustainability refers to the ability of current generations to meet their needs without limiting the ability of future generations to meet their needs. Students will investigate a range of sustainability issues, including quality of life, community resource distribution, educational attainment, and waste/recycling methods.

These investigations encourage students to build on their mathematics knowledge while simultaneously using 21st-century skills, such as critical thinking, collaboration with peers, and applying a global perspective. These combined skills will help prepare students for the future and engage them in problem-solving activities similar to ones they will encounter as active, engaged citizens.

The teacher’s guide includes 15 lessons that were inspired, researched, designed, and field tested by mathematics teachers and other education professionals. Lessons are aligned with learning standards developed by the National Council of Teachers of Mathematics. Each lesson has additionally been aligned with concepts taught in the nation’s most widely used textbooks on introductory algebra and geometry. Real world data sets and teaching ideas have been included to further support contextual math explorations.

Action projects and lesson extensions have also been included to allow learning beyond one-day classroom activities. In addition, there are master pages that correspond to the student workbook—a resource that includes lesson handouts, supplemental worksheets, and topical sustainability readings. These lessons can be used as hook activities to start a unit of study, as reinforcement for previously learned concepts, or as real world assessments of learning.

This guide is a tool to help support your efforts in equipping your students with the knowledge and skills they need to solve problems big and small. Enjoy these investigations!
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Number Patterns

Waste & Recycling

Students explore number patterns related to the disposal and recycling of plastic bottles. Long-term trends in waste production per person are graphed, and students break down the percentages of various materials that compose our waste stream.
Critical Thinking Questions
• What are historic and current recycling trends in the United States?
• What is the environmental impact of waste disposal?
• What are ways to prevent the negative impacts of waste produced in the United States?

Objectives
• Identify number patterns
• Construct expressions that contain variables to represent real-world patterns
• Use tables and graphs to organize data
• Explore patterns of waste disposal and recycling in the United States

Key Concepts
• Number patterns
• Expressions
• Tables
• Line and circle graphs
• Analysis of waste materials in the United States
• U.S. recycling rates

NCTM Standards and Expectations Addressed
Number and Operations: Work flexibly with fractions, decimals, and percents to solve problems

Number and Operations: Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods

Algebra: Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules

Algebra: Develop an initial conceptual understanding of different uses of variables

Problem Solving: Solve problems that arise in mathematics and in other contexts

Reasoning and Proof: Make and investigate mathematical conjectures

Connections: Recognize and apply mathematics in contexts outside of mathematics

Representation: Use representations to model and interpret physical, social, and mathematical phenomena

Materials/Preparation
Place the following products into a trash bag: glass bottle, plastic cup, newspaper, coffee grounds in a Ziploc bag, a piece of cardboard, an aluminum can, and a rechargeable battery.

Student handout: Waste Not, Want Not
Teacher master: Waste Not, Want Not

Background and Practice
Background reading: What’s the Big Deal?
Practice worksheet: Practice with Patterns
Teacher Instructions

1. Break the class into groups of 2-3.

2. Hold up the trash bag with the items you placed in it. Tell students that you were thinking of throwing these items into the garbage. Allow each group to choose 1 item from the bag and then tell students to work together in their group to answer the following questions: 1) Where does it go after it is used? 2) Can it be reused? If so, in what way?

3. After students brainstorm for a couple of minutes, give them the following information: a glass bottle can be used over and over and it can be recycled into other glass materials; different types of plastic can be recycled to make carpet, fleece, and many other items; newspapers can be recycled into paper, tissue, and new newspapers; coffee grounds can be used as fertilizer for plants; a cardboard box can be reused or recycled into a new cardboard box; aluminum can be recycled over and over to create new cans; and a rechargeable battery can be used in multiple products.

4. Ask students where they think all of this waste would end up if you were to throw the products into the garbage can. *Items thrown in the trash are often transported to a landfill where they remain intact for a very long time, or the trash is burned in an incinerator and the ash is sent to a landfill. Some items may travel to a recycling facility.*

5. Ask students if they think it is okay for all of these products to be in a landfill. Ask for a few volunteers to share their reasoning.

6. Tell the class that plastic bottles can be, and often are, recycled. Ask students how many soda and water bottles they use each day. Ask them to guess how many bottles are used in the United States every 5 minutes. *(2 million!)*


8. This activity can be done individually or in pairs. If this activity is used as a hook or introduction to recognizing and working with number patterns, you may want to work through the lesson together as a class.

9. After students complete the worksheet, lead a class discussion using the following questions.

Discussion Questions

1. Do you think it is important to recycle as much as we can? Why or why not?

2. In addition to recycling, what are some other ways to reduce the amount of waste we produce?
3. Do you know where the closest landfill to your house is? Would you want a landfill near your home? Why or why not? What factors might influence where landfills are located?

4. Do you think the amount of waste generated per person in the United States will continue to increase, as it has over the past 40 years? Why or why not?

5. What kinds of materials are recyclable in your community? Do you recycle these items? If not, what are the barriers to you recycling them and how can you overcome those barriers?

6. Can you think of any incentives or programs that would encourage more people to recycle?

Extension Ideas

1. Fourteen recycled 20-ounce plastic bottles can be used to make 1 XL t-shirt. If you recycled 115, how many XL t-shirts could you make? (8, with a few bottles left over)


Additional Resources


- www.thinkoutsidethebottle.org—“Think Outside the Bottle” is a campaign led by Corporate Accountability International that challenges the bottled water industry.

- www.chrisjordan.com—Photographer Chris Jordan creates visual representations of global consumption. In one photo (www.chrisjordan.com/current_set2.php), viewers can see what 2 million plastic bottles, the number used in the United States every 5 minutes, actually look like.

Action Project

Have students find alternative uses for materials currently being thrown in landfills (e.g., tires, cell phones, and computers). Then organize a campaign to support the reuse of at least one of these items and teach others at school how to do so. The campaign might include posters, video and radio announcements, and/or distribution of information in the cafeteria or at a student assembly.
Investigations

1. Did you know that 2 million plastic bottles are used every 5 minutes in the United States? Review the following data and complete the table, using the pattern shown.

Number of Bottles Used Every 5 Minutes in the United States

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<tr>
<th>Time (in minutes)</th>
<th>5</th>
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<th>15</th>
<th>20</th>
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<th>40</th>
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<th>50</th>
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<td>Bottles (in millions)</td>
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<td>4</td>
<td>6</td>
<td>8</td>
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2. How many plastic bottles are used in 1 day?

3. If \( \frac{1}{3} \) of plastic bottles are recycled daily in the United States, how many bottles each day are NOT recycled?

4. What might be 2 potential consequences, either positive or negative, of NOT recycling plastic bottles?

5. Create an equation to represent the number of bottles recycled for any number of hours, using \( b \) to represent the number of bottles.

6. Now look at the amount of waste generated per person in a single day in the United States. Organize the data in the table below on a line graph.

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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pounds per Person per Day</td>
<td>2.7</td>
<td>3.3</td>
<td>3.7</td>
<td>4.5</td>
<td>4.6</td>
</tr>
</tbody>
</table>

7. Using your graph, predict the number of pounds per day the average person will discard in 2020.

ANSWERS

1. See Table.

2. \( \frac{24 \text{ bottles}}{1 \text{ hour}} \times 24 \text{ hours} = 576 \text{ million} \)

3. \( \frac{2}{3} \times 576 = 384 \text{ million} \)

4. Possible answers could include: Bottles are going into landfills and taking a long time to decay. Chemicals from the bottles are going into the soil and ground. Littered bottles make their way to oceans instead of landfills. Petroleum used to make the bottles is not a renewable resource. Both recycling and trash disposal employ people.

5. \( b = \frac{24 \text{ million} \times h}{3} \)

6. See line graph.

7. The amount of trash discarded per person continues to increase, though the rate of increase from 1990 and 2000 is very small. A reasonable estimate might be somewhere between 4.6 and 5.5 pounds.
8. Look at the following circle graph of the materials we discard. A circle graph shows how different parts of a whole are divided.

![Circle Graph of Materials in Waste Stream]

If all materials other than plastics and food scraps = 76% of our waste stream and the percentage of plastics is equal to the percentage of food scraps, what percentage of our waste stream is plastics?

9. Recycling 4,050 20-ounce plastic bottles saves 1 cubic yard of landfill space. How many plastic bottles would need to be recycled to save 50 cubic yards of landfill space?

10. Recycling materials is one way to combat climate change. In 2006, 7 million metric tons of metals were recycled, eliminating greenhouse gas emissions equivalent to 6.5 million metric tons of carbon dioxide. (Carbon dioxide is a greenhouse gas that leads to warmer temperatures on Earth.)

If we tripled the amount of metals we recycle, how many million metric tons of carbon dioxide will be eliminated?

**Bonus**

In 2006, 32.5% of all solid waste was recycled, amounting to 81.8 million tons of recycled materials. What was the total amount of solid waste generated in 2006 (including recycled and discarded materials)? Report your answer in millions of tons.

**Answers**

8. 100% – 76% = 24%
   
   24% ÷ 2 = 12%

9. 4050 x 50 = 202,500

10. 6.5 million metric tons x 3 = 19.5 million metric tons of carbon dioxide

**Bonus**

81.8 million tons = 0.325x; x = 251.69 million tons
1. Review and complete the following tables.

   **Table I:** 2 million plastic bottles are used every 5 minutes in the United States. Complete the following table to see the pattern of bottle use over time:

<table>
<thead>
<tr>
<th>Time (in minutes)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottles (in millions)</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

   **Table II:** Now consider a different pattern, and complete the following table:

<table>
<thead>
<tr>
<th>Time (in minutes)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottles (in millions)</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>32</td>
<td>64</td>
</tr>
</tbody>
</table>

2. What is happening in Table II?

3. Why might bottle usage in the future follow the pattern in Table II rather than the pattern in Table I?

4. Data from Tables I and II are shown on the graph below. Complete the graph using the numbers you added to Table II. The resulting double bar graph will allow you to compare steady growth (Table I) to exponential growth (Table II).

   **Exponential growth** is demonstrated by the pattern in Table II, where usage is increasing faster and faster over time.

ANSWERS

1. See tables.
2. Bottle use is doubling every 5 minutes.
3. More people may use plastic bottles, or each person currently using plastic bottles may use more bottles in the future.
4. See graph.
Introduction to Algebra

Poverty & Microcredit

Through a simulation activity, students learn about microcredit lending—a financing structure that supports small-scale business ventures and is designed to help people rise out of poverty. Students solve algebraic equations in order to complete a microcredit loan application.
Critical Thinking Questions
• What are some solutions to ending extreme poverty?
• What is microcredit and how can it help alleviate poverty?

Objectives
• Evaluate formulas for different values
• Solve multistep equations
• Consider what it means to live in extreme poverty
• Brainstorm solutions to extreme poverty
• Investigate microcredit as a potential solution to extreme poverty

Key Concepts
• Distributive property
• One-step equations
• Rational numbers
• Global poverty
• Microcredit

NCTM Standards and Expectations Addressed

Number and Operations: Use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals

Algebra: Develop an initial conceptual understanding of different uses of variables

Algebra: Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships

Problem Solving: Solve problems that arise in mathematics and in other contexts

Communication: Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Connections: Recognize and apply mathematics in contexts outside of mathematics

Materials/Preparation
Table: How Far Does Your Dollar Go?, photocopied onto an overhead or projected with a document camera
Student handout: Microcredit Business Plan, 1 for each pair of students
Teacher master: Microcredit Business Plan

Background and Practice
Background reading: Poverty by the Numbers
Practice worksheet: Practice with Algebra
Teacher Instructions

1. List the following items on the board: movie ticket, bus fare, cup of coffee, and three-course dinner.

2. Ask students to estimate how much they would pay for the different items. Also, ask if they think they would be able to purchase any of the items for $2 or less.

3. Reveal the prices of the items to the students (based on New York City prices): movie ticket—$12, bus/subway fare—$2, cup of coffee—$1.60, and three-course dinner—$24.¹

4. Students could compare these prices to where they live because costs will vary across the United States.

5. Show the class the table, How Far Does Your Dollar Go? Ask them to identify the cities where they could buy each item for the lowest price. Also, which city is the most expensive?
   - Optional: Have a map available if students are unclear of where each city is located.

6. Give them a few moments to discuss with a partner whether they think it would be possible to live in any of these cities for less than $2 a day.

7. Ask volunteers to share their thoughts on the feasibility of living on less than $2 a day in any of the selected cities. Also ask students to share anything that surprises them about the different prices around the world.

8. Ask students to guess what percentage of the world’s people live on the equivalent of U.S.$2 a day or less. (About 40% of the world’s people live on $2 a day or less. Of these, approximately half survive on $1.25 a day or less.)² How would you envision the daily life of a person who lives on less than $2 a day? What kind of work might they do? What kind of food might they eat?

9. In a think-pair-share activity, ask students to brainstorm how the number of people living in extreme poverty could be reduced. Allow about 2-3 minutes for this activity. Ask for volunteer groups to share their ideas with the class.

10. Now let students know that one tool used to pull people out of extreme poverty is microcredit, which involves lending very poor people small amounts of money to start a new business or expand an existing business.

11. Split the class into pairs to complete the handout, Microcredit Business Plan.
12. Conclude the lesson using the following discussion questions or continue the lesson with one of the lesson extension ideas.

Discussion Questions

1. Imagine if you had to survive on $2 a day or less. How do you think you would spend your two dollars?

2. What business would you start if you were given a microcredit loan in the United States?

3. Does the process of microlending seem like a good way to alleviate poverty? Why or why not?

4. What do you think might be potential flaws or problems associated with microcredit?

5. What kinds of things could you do personally to help alleviate poverty, either in your local community or elsewhere in the world?

Extension Ideas

1. The 2007 poverty threshold for an individual with no children in the United States is approximately $10,500. How much money would an individual with an annual income of $10,500 make in 1 week? ($201.37) Calculate a weekly budget for this income level using the following categories: food, rent, electricity, water, transportation, and entertainment. What might happen if a person at this income level had to pay $200 to visit an emergency room? Do you think the percentage of income spent on food and housing costs would be higher or lower for a person who made more money? Why?

2. Have students split into groups and apply for a microcredit loan of $3,000 to start up a small business anywhere in the world. Have them create a loan application that outlines where their business would be located, what they would sell, how the profits would be used, and how they would sustain the business. Then have them present this information to a group of students acting as a panel of bank lenders.

Additional Resources

- *The End of Poverty?* Cinema Libre Studio, 2008, [www.theendofpoverty.com](http://www.theendofpoverty.com). This documentary explores the question: “In a world with so much wealth, why is there still so much poverty?” In the 3-minute trailer, students hear perspectives from the North and South on why global poverty persists.
• **www.grameen-info.org**—Find out more about microfinance from the Grameen Bank, the world’s original microlending organization. You can also read about the Bank’s founder, Muhammad Yunus, who was awarded the Nobel Peace Prize for his work on microcredit.

• **http://kiva.org**—This website lets people become lenders to actual microcredit projects. Once the lender selects the microcredit project they want to support, money is applied directly to this project. The lender may either withdraw the money once it is repaid or the money may be reinvested in a new project.

**Action Project**

Organize a “Penny Challenge” at your school to support a microcredit organization (such as Kiva.org, Grameen Bank, or Trickle Up) or any organization working to reduce poverty. Each penny is worth a point, and the team that gets the most points wins. Bills or coins other than pennies count as negative points (a quarter is −$0.25$ points) and can be put in the other team’s container, thus decreasing your competitor’s total points but increasing the total money raised. Display the pennies in large clear containers so that people can watch the money add up. Try competitions between boys and girls, grade levels, or different classrooms. The winning team gets to decide which organization they will donate the money to.
### How Far Does Your Dollar Go? Costs of Items around the World

<table>
<thead>
<tr>
<th>City</th>
<th>Movie Ticket</th>
<th>Bus Fare</th>
<th>Cup of Coffee</th>
<th>Three-Course Dinner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accra, Ghana</td>
<td>$10.79</td>
<td>$0.43</td>
<td>$2.15</td>
<td>$14.03</td>
</tr>
<tr>
<td>San Sebastián, Spain</td>
<td>$7.93</td>
<td>$1.45</td>
<td>$1.71</td>
<td>$19.82</td>
</tr>
<tr>
<td>São Paulo, Brazil</td>
<td>$6.57</td>
<td>$0.93</td>
<td>$0.94</td>
<td>$11.05</td>
</tr>
<tr>
<td>St. Petersburg, Russia</td>
<td>$5.74</td>
<td>$0.49</td>
<td>$0.93</td>
<td>$9.56</td>
</tr>
<tr>
<td>Mumbai, India</td>
<td>$3.40</td>
<td>$0.10</td>
<td>$0.96</td>
<td>$11.35</td>
</tr>
<tr>
<td>Melaka, Malaysia</td>
<td>$2.50</td>
<td>$0.67</td>
<td>$0.58</td>
<td>$15.68</td>
</tr>
<tr>
<td>Kathmandu, Nepal</td>
<td>$2.75</td>
<td>$0.20</td>
<td>$0.91</td>
<td>$7.98</td>
</tr>
<tr>
<td>Bismarck, USA</td>
<td>$7.75</td>
<td>$1.00</td>
<td>$1.00</td>
<td>$20.00</td>
</tr>
</tbody>
</table>
1. Calculate your total operational costs (TOC) using the following equation, where \( n \) = number of months: \( \text{TOC} = Sc + n \times Mc \)

<table>
<thead>
<tr>
<th>Number of Months</th>
<th>Total Operational Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 800 + 1(1,300) = 2,100 ) rupees</td>
</tr>
<tr>
<td>6</td>
<td>8,600 rupees</td>
</tr>
<tr>
<td>12</td>
<td>16,400 rupees</td>
</tr>
</tbody>
</table>

2. Calculate your profit (P) for a given period of time using the following equation, where \( S \) = monthly sales, \( Mc \) = monthly costs, and \( n \) = number of months: \( P = n(S - Mc) \)

<table>
<thead>
<tr>
<th>Number of Months</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 1(3,500 - 1,300) = 2,200 ) rupees</td>
</tr>
<tr>
<td>6</td>
<td>13,200 rupees</td>
</tr>
<tr>
<td>12</td>
<td>26,400 rupees</td>
</tr>
<tr>
<td>18</td>
<td>39,600 rupees</td>
</tr>
</tbody>
</table>

3. What do you notice about the profit in the given amount of time that you calculated? Explain your thinking in 2-3 sentences.

4. Convert the number of rupees made in 1 year to U.S. dollars.

ANSWERS

1. See table.
2. See table.
3. Possible answers could include: The profit increases linearly from month 1 (2,200 rupees) to month 18 (39,600 rupees). This is an 18-fold increase. This is a significant increase that could help support someone living in poverty.
4. 26,400 rupees/43 rupees = approximately U.S.$613
5. What is your expected net profit after 6 months?

(Hint: To find the net product, subtract your start-up costs)

6. **Interest** is the money that a lender charges to borrowers; it is usually a percentage of the loan amount. When repaying a loan, borrowers must pay interest in addition to the principal loan amount.

For a principal loan amount of 2,000 rupees loaned at an annual interest rate of 20%, calculate how much interest (I) you will need to pay back for different periods of time. Use the following equation, where \( p = \) principal loan amount, \( r = \) interest rate, and \( t = \) time in years: 

\[
I = p \times r \times t
\]

(Hint: You will need to convert the interest rate, 20%, into the decimal 0.20 to solve the equation)

<table>
<thead>
<tr>
<th>Number of Years</th>
<th>Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( I = (2,000)(0.20)(1) = 400 ) rupees</td>
</tr>
<tr>
<td>2</td>
<td>800 rupees</td>
</tr>
<tr>
<td>3</td>
<td>1,200 rupees</td>
</tr>
</tbody>
</table>

**Bonus**

A lending organization has raised $4,000 that it wants to use to provide $100 microloans to people in one community. Of the $4,000 raised, $500 must be used for program administration costs (to pay for paperwork and salaries for people who will help set up the loans). How many microcredit loans can be provided with the organization's $4,000?

\[
100x + 500 = 4,000; \quad x = 35 \text{ people}
\]
Algebra is a branch of mathematics that uses letters to represent variables, or unknown values. The following algebraic equation includes the variable, \( x \).

Example: \( 3x = 42 \)

Solving an **algebraic equation** allows us to find the value of the variable. How would you solve the previous equation?

You might solve it with mental math. Or you might use inverse operations. For example, to isolate \( x \) from 3, you could divide both sides by 3.

\[
\frac{3x}{3} = \frac{42}{3} \\
x = 14
\]

Sandra lives in Guatemala. She is applying for a microcredit loan to start a street food business. **Microcredit** is a system of lending people very small amounts of money. Complete the following calculations to determine Sandra’s anticipated costs and profits.

1. Start-up costs for the business include buying a cart for selling the food on the street, a freezer, and bulk foods. The cart costs $65, and the freezer costs $900. The total start-up cost is $1053. Write an equation to represent the start-up costs, assigning a variable for the bulk foods.

2. Solve your equation to find the cost of the bulk foods Sandra plans to buy.

3. Sandra calculates her monthly costs (\( C \)) to be $415, and she expects monthly sales (\( S \)) to be $700. Calculate her profit (\( P \)) for a 6-month period using the following equation, where \( n \) represents the number of months:

   \[
P = n(S - C)
   \]

4. Sandra wants to use a portion of her profits to invest in community health care projects, including funding health education and a community clinic. She decides to give $125 each month to health care projects. Which of the following equations best represents this situation?
   
   a) \( P = n(S - 125 - C) \)  
   b) \( P = n(S - C + 125) \)  
   c) \( P + 125 = n(S - C) \)

5. Explain why the other 2 equations are incorrect.

6. A microcredit loan business is so impressed by Sandra’s business plan that they decide to give her an additional $100 per month to include in her monthly sales. What is the amount of sales she will now be able to earn in 1 year?

### ANSWERS

1. \( 65 + 900 + b = 1053 \), or \( 965 + b = 1053 \)  
2. \( 965 + b = 1053 \)  
   
   \( b = 88 \)  
3. \( P = 6(700 - 415) \)  
   
   \( P = 1710 \)  
4. a

5. Equation b adds $125, when the net profit should be reduced by $125. If \( (C + 125) \) were in parentheses, the equation would be correct.  
   
   Equation c adds $125 to the profit.

6. monthly sales = $700 + $100 = $800  
   
   \( 800 \times 12 \) months = $9600
Modeling Integers

Population Growth

Students examine growing and declining populations and predict future population growth based on current population growth rates. Integers are plotted on a coordinate plane to examine the pattern of population decline in Japan. Students discuss the consequences of different patterns of population growth.
Critical Thinking Questions
• What factors drive population growth?
• What are some impacts of population growth and population loss?
• How is population connected to other global issues?

Objectives
• Recognize and identify integers
• Add integers with same and different signs
• Solve equations with integers
• Plot integers on a coordinate plane
• Define population growth rate
• Examine population trends
• Consider some consequences of positive and negative population growth

Key Concepts
• Integers
• Plotting points on a coordinate plane
• Population growth trends

NCTM Standards and Expectations Addressed
Number and Operations: Develop meaning for integers and represent and compare quantities with them
Number and Operations: Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers
Algebra: Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules

Algebra: Develop an initial conceptual understanding of different uses of variables
Algebra: Model and solve contextualized problems using various representations, such as graphs, tables, and equations
Data Analysis and Probability: Select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots
Problem Solving: Solve problems that arise in mathematics and in other contexts
Communication: Communicate mathematical thinking coherently and clearly to peers, teachers, and others
Connections: Recognize and apply mathematics in contexts outside of mathematics
Representation: Use representations to model and interpret physical, social, and mathematical phenomena

Materials/Preparation
Cards: Children Born per 100 Women by Country, 1 per student
(Optional) Blank sheets of 8.5” x 11” paper, 1 per student
Student handout: The Ups and Downs of Population, 1 per pair
Teacher master: The Ups and Downs of Population
Graph paper, 1 page per pair

Background and Practice
Student reading: Increasing Numbers
Practice worksheet: Practice with Integers
Teacher Instructions

1. Briefly introduce or review integers. Integers are the set of positive and negative whole numbers and zero. For example, $-5, 0, \text{ and } 5$ are all integers.

2. Ask students if they can think of some real-life examples of positive integers. If they need prompting, ask them the following questions: How old are you? How many classes do you have in a school day? How many players from each team can play on a basketball court at one time?

3. Ask students to think of examples of negative integers. Negative temperatures, land below sea level, and money owed are just a few examples.

4. Introduce the link between integers and population growth by asking the class, “Can you have a negative number of people?” (No, but you can have negative population growth.)

5. Pass out 1 Children Born per 100 Women by Country card to each student. Tell them that the number written on the card is the number of children born per 100 women in that country. Ask for a volunteer to tell you how you could figure out the average number of children born per each woman. (Divide the given number by 100.)

6. Ask students to arrange themselves into a human number line from largest to smallest number of children born per 100 women.

7. Have students form a semi-circle so that each card is visible to everyone, or have students read their country and number aloud one at a time.

8. Ask the class to consider the consequences if all countries had the same number of children as Mali (7.34 per woman). What about Singapore (1.08 per woman)?

9. Ask students to make observations about why they believe certain countries have a higher birthrate than other countries.

10. Share the following definition of population growth rate with students or ask them to come up with their own definition. Population growth rate is the change in a population’s size over time. If a population is getting larger over time, it has a positive population growth rate. If a population is getting smaller over time, it has a negative growth rate.

12. After student pairs have completed the worksheet, review population trends in Japan and India using the following discussion questions.

**Discussion Questions**

1. Based on the projected population numbers for Japan in 2050, what do you think Japan’s population size will be in 2100?

2. What are some negative consequences of a declining population?

3. What are some negative consequences of a growing population?

4. Do you think the future populations of India and Japan will be identical to the numbers shown on the population pyramids? Why or why not?

5. What are some humane ways to slow population growth?

**Extension Ideas**


   How will population density (people per area) change over time in Japan? In India?

   What might be some consequences of higher population density? *(e.g., conflict, resource scarcity, migration)*

2. Investigate the population growth rate in the United States (see [www.census.gov](http://www.census.gov)). Compare birth rates to growth rates. What other reasons might explain population growth in the United States? *(e.g., immigration)*

**Additional Resource**

[www.prb.org](http://www.prb.org) — Population Reference Bureau publishes an annual World Population Data Sheet, which reports on population-related statistics by country. The data sheet includes information related to demographics, nutrition, environment, economy, and density. In addition, PRB has many downloadable population graphics.

**Action Project**

While the United States makes up only 5% of the world’s population, its citizens consume 25% of the earth’s energy resources. The energy consumed in our homes and our methods of transportation are the two biggest culprits. Have each student make a pledge to reduce his or her impact in one of these areas for 1 week. Share students’ results, including the unforeseen difficulties or obstacles they encountered when trying to reduce energy consumption.
## Children Born per 100 Women by Country

<table>
<thead>
<tr>
<th>Country</th>
<th>Region</th>
<th>Children Born per 100 Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mali</td>
<td>Africa</td>
<td>734</td>
</tr>
<tr>
<td>Malaysia</td>
<td>Southeast Asia</td>
<td>298</td>
</tr>
<tr>
<td>Algeria</td>
<td>Africa</td>
<td>182</td>
</tr>
<tr>
<td>Burkina Faso</td>
<td>Africa</td>
<td>634</td>
</tr>
<tr>
<td>Israel</td>
<td>Middle East</td>
<td>277</td>
</tr>
<tr>
<td>Norway</td>
<td>Europe</td>
<td>178</td>
</tr>
<tr>
<td>Mauritania</td>
<td>Africa</td>
<td>569</td>
</tr>
<tr>
<td>Nicaragua</td>
<td>Central America</td>
<td>263</td>
</tr>
<tr>
<td>Seychelles</td>
<td>Indian Ocean, east of Africa</td>
<td>173</td>
</tr>
<tr>
<td>Chad</td>
<td>Africa</td>
<td>543</td>
</tr>
<tr>
<td>Bahrain</td>
<td>Middle East</td>
<td>253</td>
</tr>
<tr>
<td>Netherlands</td>
<td>Europe</td>
<td>166</td>
</tr>
<tr>
<td>Madagascar</td>
<td>Indian Ocean, east of Africa</td>
<td>519</td>
</tr>
<tr>
<td>Faroe Islands</td>
<td>North Atlantic Ocean, near Europe</td>
<td>245</td>
</tr>
<tr>
<td>Cuba</td>
<td>Caribbean Sea</td>
<td>160</td>
</tr>
<tr>
<td>Togo</td>
<td>Africa</td>
<td>485</td>
</tr>
<tr>
<td>Indonesia</td>
<td>Southeast Asia</td>
<td>234</td>
</tr>
<tr>
<td>Portugal</td>
<td>Europe</td>
<td>149</td>
</tr>
<tr>
<td>Sudan</td>
<td>Africa</td>
<td>458</td>
</tr>
<tr>
<td>New Caledonia</td>
<td>Pacific Ocean, Oceania</td>
<td>221</td>
</tr>
<tr>
<td>Russia</td>
<td>Eastern Europe/Asia</td>
<td>140</td>
</tr>
<tr>
<td>Iraq</td>
<td>Middle East</td>
<td>397</td>
</tr>
<tr>
<td>Azerbaijan</td>
<td>Eastern Europe/Asia</td>
<td>205</td>
</tr>
<tr>
<td>Hungary</td>
<td>Europe</td>
<td>134</td>
</tr>
<tr>
<td>Papua New Guinea</td>
<td>Pacific Ocean, Oceania</td>
<td>371</td>
</tr>
<tr>
<td>France</td>
<td>Europe</td>
<td>198</td>
</tr>
<tr>
<td>Slovenia</td>
<td>Europe</td>
<td>127</td>
</tr>
<tr>
<td>American Samoa</td>
<td>Pacific Ocean, Oceania</td>
<td>335</td>
</tr>
<tr>
<td>Uruguay</td>
<td>South America</td>
<td>194</td>
</tr>
<tr>
<td>Japan</td>
<td>Pacific Ocean, East Asia</td>
<td>122</td>
</tr>
<tr>
<td>Lesotho</td>
<td>Africa</td>
<td>313</td>
</tr>
<tr>
<td>Lebanon</td>
<td>Middle East</td>
<td>187</td>
</tr>
<tr>
<td>Singapore</td>
<td>Southeast Asia</td>
<td>108</td>
</tr>
</tbody>
</table>
Investigations

1. Answer the following questions to see what you already know about population growth.
   a. Do you think most countries have growing or declining populations?
   b. Name 1 thing that might lead to a population increase.
   c. Name 1 thing that might cause population to decline.

2. Look at the age-gender structure of Japan in 2000:¹

<table>
<thead>
<tr>
<th>Age</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–9</td>
<td>12,142,579</td>
<td>6,627,109</td>
</tr>
<tr>
<td>10–19</td>
<td>14,057,715</td>
<td>7,412,041</td>
</tr>
<tr>
<td>20–29</td>
<td>18,580,447</td>
<td>8,605,113</td>
</tr>
<tr>
<td>30–39</td>
<td>16,776,034</td>
<td>8,605,113</td>
</tr>
<tr>
<td>40–49</td>
<td>16,789,176</td>
<td>8,605,113</td>
</tr>
<tr>
<td>50–59</td>
<td>19,041,467</td>
<td>11,665,199</td>
</tr>
<tr>
<td>60–69</td>
<td>14,679,053</td>
<td>12,738,388</td>
</tr>
<tr>
<td>70–79</td>
<td>9,890,833</td>
<td>14,578,186</td>
</tr>
<tr>
<td>80+</td>
<td>4,771,919</td>
<td>13,444,661</td>
</tr>
</tbody>
</table>

3. Use the information provided in the following table to calculate the number of people in each age group in 2050. Use the following equation to find Population Totals for 2050, where $P_{2050}$ is Population Totals for 2050, $P_{2000}$ is Population Totals for 2000, and $C$ is Change in Population from 2000 to 2050:

   $$P_{2050} = P_{2000} + C$$

<table>
<thead>
<tr>
<th>Age</th>
<th>Population Totals for 2000</th>
<th>Change in Population from 2000 to 2050</th>
<th>Population Totals for 2050</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–9</td>
<td>12,142,579</td>
<td>−5,515,470</td>
<td>6,627,109</td>
</tr>
<tr>
<td>10–19</td>
<td>14,057,715</td>
<td>−6,645,674</td>
<td>7,412,041</td>
</tr>
<tr>
<td>20–29</td>
<td>18,580,447</td>
<td>−10,641,656</td>
<td>7,938,791</td>
</tr>
<tr>
<td>30–39</td>
<td>16,776,034</td>
<td>−8,170,921</td>
<td>8,605,113</td>
</tr>
<tr>
<td>40–49</td>
<td>16,789,176</td>
<td>−6,124,838</td>
<td>10,664,338</td>
</tr>
<tr>
<td>50–59</td>
<td>19,041,467</td>
<td>−7,376,268</td>
<td>11,665,199</td>
</tr>
<tr>
<td>60–69</td>
<td>14,679,053</td>
<td>−1,940,665</td>
<td>12,738,388</td>
</tr>
<tr>
<td>70–79</td>
<td>9,890,833</td>
<td>+4,687,353</td>
<td>14,578,186</td>
</tr>
<tr>
<td>80+</td>
<td>4,771,919</td>
<td>+8,672,742</td>
<td>13,444,661</td>
</tr>
</tbody>
</table>

4. Add the Change in Population from 2000 to 2050 numbers for ages 50–80+. Based on your answer, how will the number of people older than 50 change from 2000 to 2050?

5. Which 10-year age group will lose the greatest number of people from 2000 to 2050?

6. What is 1 way that the predicted population structure for 2050 might affect Japan?

   See table.

   There will be an additional 4,043,162 people in this age range.

   20–29

   Answers might include reduced population density (i.e., more land and resources for each person), fewer schools needed, or more elderly people to be supported.
7. Look at the age-gender structure predicted for India in 2050. Describe in 1-2 sentences how the population in India will differ from Japan’s population in 2050.

8. What is 1 way that the predicted population structure for 2050 might affect India?

9. Japan’s population growth rate is $-0.139\%$.
What integer is closest to the numeric value of $-0.139\%$?

ANSWERS
7. India has a much larger, younger population than Japan.
8. Answers might include more competition for resources (land, food, water), crowded cities, more pressure on the natural environment, or greater political and cultural influence in the world.
9. 0

10. Plot the coordinate pairs in the following table on a coordinate plane in order to see a visual representation of how Japan’s population is changing. The $x$-values are 5-year time intervals. The $y$-values represent changes in population in millions of people, rounded to the nearest million.

<table>
<thead>
<tr>
<th>Time ($x$)</th>
<th>Change in Population ($y$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>0</td>
</tr>
<tr>
<td>1995</td>
<td>2</td>
</tr>
<tr>
<td>2000</td>
<td>3</td>
</tr>
<tr>
<td>2005</td>
<td>4</td>
</tr>
<tr>
<td>2010</td>
<td>3</td>
</tr>
<tr>
<td>2015</td>
<td>1</td>
</tr>
<tr>
<td>2020</td>
<td>-3</td>
</tr>
<tr>
<td>2025</td>
<td>-7</td>
</tr>
<tr>
<td>2030</td>
<td>-11</td>
</tr>
<tr>
<td>2035</td>
<td>-16</td>
</tr>
<tr>
<td>2040</td>
<td>-21</td>
</tr>
<tr>
<td>2045</td>
<td>-26</td>
</tr>
<tr>
<td>2050</td>
<td>-31</td>
</tr>
</tbody>
</table>

11. What is the trend of the graph you created? Explain your observations in 1-2 sentences.

Bonus
Given that India’s population growth rate is $+1.578\%$, create an algebraic equation to determine future population size in India, where $x$ represents the current population and $y$ represents the future population. Then solve your equation for $x = 1$ billion.

11. Population size increases from 1990 until 2005 and then declines at a fairly steady rate through 2050.

Bonus
$y = x + 0.01578x$, or $y = 1.01578x$
If $x = 1$ billion, $y = 1,015,780,000$
Positive and negative whole numbers and 0 are all **integers**.

**Absolute value** of any number is the distance between that number and 0 on the number line. For example, both -4 and 4 are the same distance from 0 on the number line.

So the absolute value of both -4 and 4 is 4. Absolute value is indicated by 2 vertical bars on either side of a number: \(|-4|\) and \(4|\)

1. Consider a place called Sunspot Island. Each year the population doubles, or multiplies by 2. Complete the following table to show the growth of the island’s population over a 10-year period.

<table>
<thead>
<tr>
<th>Year</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
</tr>
<tr>
<td>3</td>
<td>800</td>
</tr>
<tr>
<td>4</td>
<td>1600</td>
</tr>
<tr>
<td>5</td>
<td>3200</td>
</tr>
<tr>
<td>6</td>
<td>6400</td>
</tr>
<tr>
<td>7</td>
<td>12,800</td>
</tr>
<tr>
<td>8</td>
<td>25,600</td>
</tr>
<tr>
<td>9</td>
<td>51,200</td>
</tr>
<tr>
<td>10</td>
<td>102,400</td>
</tr>
</tbody>
</table>

2. Suppose there is only enough land on the island to grow food for 200,000 people. What will happen to Sunspot Island and its people in a very short time? Think of at least 2 possible impacts of population growth on the environmental resources and people of Sunspot.

3. In year 12, a disease begins to reduce the population of Sunspot Island. Then, in year 14, a civil war breaks out, resulting in many deaths. Perform the calculations in the middle column to discover how the population changes each year.

<table>
<thead>
<tr>
<th>Year</th>
<th>Perform These Calculations</th>
<th>Resulting Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>(-100,000(-2)) = 200,000</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>(170,000 + (-20,000)) = 150,000</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>(-240,000 ÷ (-2)) = 120,000</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>(83,500 - 8,600) = 74,900</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>(</td>
<td>2</td>
</tr>
</tbody>
</table>

4. Explain what appears to be happening to Sunspot’s population in year 15.

**ANSWERS**

1. See table.
2. If population continues to double, in year 11 the number of people will exceed the number that the island can sustain. This could lead to depletion of resources (food, water, soil), fighting over scarce resources, and starvation.
3. See table.
4. Growth has stopped. The population is neither increasing nor decreasing.
Students examine how choices in daily activities and nutrition affect individual well-being and sustainability. They consider how caloric intake relates to calories burned. They read nutrition labels, review recommended daily allowances, and solve algebraic equations to determine healthy eating habits.
Critical Thinking Questions
• What choices can individuals make to ensure a healthy lifestyle?
• How are eating habits related to future well-being?
• How is an individual’s well-being connected to the well-being of his/her community

Objectives
• Write and solve multi-step equations
• Use tables to solve real-life problems
• Recognize the connection between health, nutrition, and exercise
• Understand that daily choices can affect an individual’s well-being

Key Concepts
• Algebraic equations
• Nutrients
• Calories
• Personal health and well-being

NCTM Standards and Expectations Addressed
Number and Operations: Work flexibly with fractions, decimals, and percents to solve problems
Algebra: Develop an initial conceptual understanding of different uses of variables

Algebra: Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships
Algebra: Recognize and generate equivalent forms for simple algebraic expressions and solve linear equations
Algebra: Model and solve contextualized problems using various representations, such as graphs, tables, and equations
Problem Solving: Solve problems that arise in mathematics and in other contexts
Communication: Communicate mathematical thinking coherently and clearly to peers, teachers, and others
Connections: Recognize and apply mathematics in contexts outside of mathematics

Materials/Preparation
Student handout: You Are What You Eat, 1 per student pair
Teacher master: You Are What You Eat

Background and Practice
Background reading: What to Eat?
Practice worksheet: Practice with Solving Equations
**Teacher Instructions**

1. Have all students stand up next to their seats. Ask everyone to either do jumping jacks or run in place when you say “go.” Say “go” and time them for 10 seconds; then say “stop.”

2. Inform them that they have all just burned about \( \frac{1}{5} \) of 1 calorie.\(^1\) Ask if anyone knows what a **calorie** is or where they have heard that word before. *A calorie is a measure of heat energy. We get that energy from foods we eat.*

3. Ask students to predict how the amount of calories burned would be affected if they had exercised for more than 10 seconds. *It would increase.*

4. Ask a volunteer to define **health**. *The World Health Organization defines health as a state of complete physical, mental, and social well-being.*

5. Ask students what actions and choices they believe make an individual healthy. What are the choices an individual has to make in order to ensure optimal health? *Examples include eating nutritious foods, exercising, not using tobacco.*

6. Ask students to name some common breakfast items that they and their families like to eat. Create a list of these items where all students can see it (i.e., on a board in front of the class). Ask students which items they consider to be healthy. Put a star by these items on your list.

7. Ask the class how many high school students they think eat 5 or more servings of fruits or vegetables daily. \((20\%)\)

8. Divide the class into pairs and distribute copies of the handout, *You Are What You Eat.*

9. Have students work with a partner to complete the handout.

10. Review answers with students, and conclude the class with the following discussion questions.

**Discussion Questions**

1. What are some of the choices that Estelle needs to make in order to have a healthier lifestyle? Do any of these choices apply to your lifestyle?

2. Is everyone capable of making those choices? Why or why not?

3. How could schools help support healthier nutritional and exercise habits of students?

4. What are some ways that we could decrease U.S. childhood obesity?

5. How is personal health and well-being connected to the well-being of your community? How is the overall health of our country’s people connected to the well-being of our nation?
Extension Ideas

1. Create 3 well-balanced meals for Estelle. Look at [www.calorie-count.com](http://www.calorie-count.com) for specific food choices, caloric values, and recommended daily values.

2. A competitive swimmer eats 12,000 calories per day when he trains. Here is his daily intake when he is training:

   **Breakfast**
   - 3 fried egg sandwiches with cheese, tomatoes, lettuce, fried onions, and mayonnaise
   - 3 chocolate-chip pancakes
   - A 5-egg omelette
   - 3 slices of French toast
   - A bowl of grits
   - 2 cups of coffee

   **Lunch**
   - Half a kilogram of enriched pasta
   - 2 large ham and cheese sandwiches on white bread with mayonnaise
   - A few energy drinks

   **Dinner**
   - Half a kilogram of pasta with carbonara sauce
   - Large pizza
   - A few energy drinks

Based on his current diet, make this swimmer’s diet even healthier! Make sure he is fulfilling his 12,000 daily caloric intake while simultaneously getting the daily values of nutrients he also needs.

3. Have students bring in 3 different labels for the same type of product (cereal, potato chips, granola bars, etc.). Have them analyze the nutritional value on the labels to figure out which product is the healthiest.

Additional Resources

- [www.mypyramid.gov](http://www.mypyramid.gov)—A United States Department of Agriculture website that gives tips on how to stay healthy and even offers a menu planner based on age, gender, and physical activity of a person.

- [www.savethechildren.org](http://www.savethechildren.org)—Save the Children is an organization that works to help children in the United States and around the world. They offer multimedia stories and other resources to educate people about hunger and other issues affecting children around the world.

Action Project

Have students find out what kinds of foods are readily available in their neighborhood. They can create a list of 10 foods that are part of a healthy diet. They should think carefully about which foods are most healthy: fresh or processed fruits and vegetables, organic or nonorganic dairy products, fish or beef, etc. They can then visit the grocery store closest to their house and see if all 10 foods listed are available. If not, they can talk to the store manager about why the store should offer those foods for sale.
Investigations
Read the following case study with a partner:

Estelle M. is an eighth grade student. She plays on the basketball team and takes great pride in being the point guard. Recently, she hasn’t been playing to her best ability. She has felt extremely tired and can’t seem to concentrate in class. Even during practice, she is lethargic. This makes her nervous because the team has the potential to win the championships. To win the championships, everyone, including Estelle, needs to be playing her best.

Estelle’s mom decides to take her to the doctor. The doctor tells her that her participation in strenuous exercise is great, but he worries that she isn’t eating enough calories to support her high activity level. Her doctor asks her to keep track of her caloric intake and the amount of basketball she plays daily.

1. Estelle weighs 150 pounds, and she needs a minimum of 2,560 calories per day to maintain her weight. Her doctor explained that when she plays full court basketball, she burns extra calories. Therefore, she should consume more than 2,560 calories on days when she plays full court basketball. Her doctor suggested that she consume about 11 extra calories for each minute of strenuous exercise.

Create an equation that Estelle can use to determine how many total calories she needs to eat in a day for a certain amount of exercise.

2. Yesterday, Estelle played basketball for 2 hours. She ate 3,100 calories worth of food. Did Estelle consume enough calories?

3. If Estelle plays basketball for 1 hour per day for the next 5 days, how many calories will she need to consume over that 5-day period? Write and solve an equation showing the number of calories Estelle needs to consume in these 5 days.

ANSWERS
1. calories = 2560 + 11m
2. calories = 2560 + 11(120 minutes) = 3880
   No, Estelle did not eat enough. She needs an additional 780 calories.
3. calories = 5[2560 + (11 × 60)]
   calories = 16,100
Estelle's doctor explains to her that he thinks she may not be consuming an adequate amount of calories and nutrients. Empty calorie foods have many calories but very few nutrients. High nutrient foods have a much greater amount of recommended daily nutrients (more than 20%), such as iron, calcium, and vitamin C.

While Estelle is exhausted by the end of practice, she has noticed that her teammate Staci still has a lot of energy. She decides to compare her lunch with Staci’s to see if Staci’s high energy might be related to the calories or nutrients she takes in.1

### Estelle's Lunch

<table>
<thead>
<tr>
<th>Item</th>
<th>Calories</th>
<th>% Daily Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamburger</td>
<td>279</td>
<td>Calcium: 6%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Vitamin C: 3%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Iron: 15%</td>
</tr>
<tr>
<td>French fries</td>
<td>539</td>
<td>Calcium: 2%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Vitamin C: 8%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Iron: 13%</td>
</tr>
<tr>
<td>Soda</td>
<td>210</td>
<td>Calcium: 0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Vitamin C: 0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Iron: 0%</td>
</tr>
<tr>
<td>1 cupcake</td>
<td>100</td>
<td>Calcium: 10%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Vitamin C: 0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Iron: 0%</td>
</tr>
</tbody>
</table>

### Staci's Lunch

<table>
<thead>
<tr>
<th>Item</th>
<th>Calories</th>
<th>% Daily Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turkey and cheese sandwich</td>
<td>361</td>
<td>Calcium: 22%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Vitamin C: 12%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Iron: 13%</td>
</tr>
<tr>
<td>Spinach salad (with tomatoes, croutons, and dressing)</td>
<td>_______ calories</td>
<td>Calcium: 27%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Vitamin C: 94%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Iron: 42%</td>
</tr>
<tr>
<td>2% milk (2 cups)</td>
<td>244</td>
<td>Calcium: ______</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Vitamin C: 1%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Iron: 0%</td>
</tr>
<tr>
<td>2 bananas</td>
<td>400</td>
<td>Calcium: 1%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Vitamin C: 33%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Iron: 3%</td>
</tr>
</tbody>
</table>

4. A serving of French fries contains 539 calories. The amount of calories in a spinach salad ($s$) can be found by solving the following equation, where $f$ is the number of calories in French fries: $s = \frac{1}{2} (f) + 16.5$.

Solve the equation to determine the number of calories in a spinach salad.

5. Staci’s lunch provides her with 108% of her daily calcium. What is the % daily value of calcium contained in the 2 cups of milk she drinks for lunch?

6. Calculate the difference in Estelle’s and Staci’s calories and percent daily values of calcium, vitamin C, and iron.

7. Based on your calculations, what are some possible reasons that Staci has more energy than Estelle at the end of practice?

8. Both Staci and Estelle are much less active during the summer, when they are not playing basketball daily. How do you think their caloric intake should be adjusted for summer months?

ANSWERS

4. $s = \frac{1}{2} (539) + 16.5$
   $s = 286$ calories

5. $22 + 27 + 1 + x = 108$
   $x = 58%$

6. Estelle: calcium = 18%, vitamin C = 11%, iron = 28%, calories = 1128
   Staci: calcium = 108%, vitamin C = 140%, iron = 58%, calories = 1291
   Staci receives 90% more calcium, 129% more vitamin C, 30% more iron, and 163 more calories than Estelle.

7. Staci eats foods that provide her with more calories and more essential nutrients. Any of those factors may be responsible for her increased energy.

8. They should eat fewer calories.
When solving an equation to find the value of a variable, you must isolate the variable, or put it by itself on one side of the equal sign. To do this, you can use inverse operations. Remember to perform the same operations on both sides of the equation. Here is an example:

\[ 3b + 24 = 42 \]
\[ 3b + 24 - 24 = 42 - 24 \quad \text{Step 1: Subtract 24 from both sides.} \]
\[ 3b = 18 \]
\[ 3b ÷ 3 = 18 ÷ 3 \quad \text{Step 2: Divide both sides by 3.} \]
\[ b = 6 \]

1. Janice runs track. In 1 day, she burned 600 calories during a strenuous practice, which was a quarter of the total calories she ate that day. Write an algebraic equation for this situation, using a variable to represent total calories.

2. Solve your equation to find the total number of calories Janice consumed.

3. Charlotte and Jesse monitor their daily intake of iron. Iron is an important nutrient that helps give your body energy for physical activity. On Monday, Charlotte consumed 3 times less iron than Jesse did. The total amount of iron they both consumed on Monday was 36 mg. Set up an equation for this situation. Then, find the amount of iron each of them consumed.

4. In many places around the world, rice is one of the main components of people’s diets. If 1 cup of rice = 205 calories, determine the amount of calories consumed in 2 separate meals that include rice (\( r \) equals the number of calories in 1 cup of rice).

   \[ 2r + 11 = \text{_____ calories in meal} \]
   \[ 4(r - 75) = \text{_____ calories in meal} \]

5. If a 175-pound man who exercises daily needs over 3000 calories to maintain a constant weight, would either of the meals in problem #4 be a good choice? Why or why not?

ANSWERS

1. \( 600 = \frac{1}{4}c \), or \( 600 = 0.25c \)
2. \( c = 2400 \) calories
3. \( J + C = 36 \) and \( J = 3C \)
   \[ C + 3C = 36 \]
   \[ C = 9 \text{ mg} \) and \( J = 27 \text{ mg} \)
4. a. 421
   b. 520
5. Meal b is better than meal a, but neither is a good choice because they do not provide enough calories. He would have to eat 7 meal a’s or 6 meal b’s to maintain his weight.
Lesson 5

Data & Graphs

Youth Conflict

Students examine information compiled by the Youth Violence Project to explore recent trends in violence among youth. Students create a variety of graphs to represent various statistics related to conflict among youth. They then reflect on possible root causes of the observed trends.
Critical Thinking Questions
• How do actual youth violence trends compare to public perceptions?
• What factors are correlated with a reduction in violence?
• What can people do to increase peace in their schools, communities, country, and world?

Objectives
• Use double bar graphs to represent data
• Use line graphs to evaluate changes in youth violence over time
• Create a scatterplot and line of best fit to determine correlation among 2 variables
• Examine trends in violent crimes among youth
• Explore possible root causes of youth violence

Key Concepts
• Bar graph
• Line graph
• Scatterplot
• Trend line
• Conflict and violence

NCTM Standards and Expectations Addressed
Data Analysis and Probability: Select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots

Data Analysis and Probability: Make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit

Problem Solving: Solve problems that arise in mathematics and in other contexts

Communication: Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Connections: Recognize and apply mathematics in contexts outside of mathematics

Representation: Use representations to model and interpret physical, social, and mathematical phenomena

Materials/Preparation
Agree and Disagree signs: In large letters, write Agree on a piece of 8.5” x 11” paper and Disagree on another piece of paper. Tape the Agree sign to 1 wall in your classroom and the Disagree sign to the opposite wall.

Graphing paper, 1 page per student pair
Ruler or other straightedge device, 1 per pair
Student handout: Increasing the Peace, 1 per pair
Teacher master: Increasing the Peace

Background and Practice
Background reading: Peace and Conflict
Practice worksheet: Practice with Data & Graphs
Teacher Instructions

1. Refer to the Materials/Preparation section for instructions regarding the Agree and Disagree signs. Write on the board the following statement: “Youth violence is a major problem in our society.”

2. Ask students to decide whether they agree or disagree with this statement. Those who agree should stand near the wall with the Agree sign. Those who disagree should stand near the wall with the Disagree sign. Ask for 2-3 volunteers from each group to explain why they agree or disagree. Then ask students to return to their seats.

3. Ask students if they think the rate of violent crimes in U.S. schools (crimes committed per 1000 students) has increased or decreased over the last 15 years. Why do they think the crime rate has increased or decreased? What have they observed in their own experience?

4. Ask them why the number of crimes per 1000 students is considered a rate. A rate is a relationship between 2 quantities with different units. In this case the quantities are crimes and students.

5. Have students complete the worksheet, Increasing the Peace, in pairs. Supply student pairs with graph paper and rulers to create their graphs.

6. Make sure that students understand that time (in years) is the independent variable for each of the graphs they will be making.

7. To shorten this activity, determine appropriate scale and axes labels for each of the 3 graphs before students graph them individually. Or, delete 1 of the graphing problems.

8. After students have observed data from the Youth Violence Project, lead a class discussion using the following questions as discussion prompts.

Discussion Questions

1. Does media portrayal of student crimes in the news represent the same trends that you discovered in this activity? If not, why do you think the media do not accurately portray national student violence trends?

2. According to data from the National Center for Education Statistics, middle school principals cite racial tension and student bullying as the most common causes of discipline problems. High
school principals, on the other hand, cite gang activities and cult or extremist group activities as the most common causes of discipline problems. Why do you think racial tension and student bullying are a bigger problem in middle schools than in high schools?

3. The percentage of students who report carrying a weapon to school is approximately twice as high for male students as it is for female students. What are possible reasons for this discrepancy?

4. A certain school district allows teachers to carry concealed firearms, provided that the teachers meet certain criteria (obtain permit to carry concealed weapon, attend crisis management training, and receive approval by the school district). Do you think that teachers should be allowed to carry concealed firearms? Why or why not?

5. What measures do you think would decrease student crime rates even further?

Extension Ideas


- violent crimes per 1000 students
- homicides on school grounds during school day
- juvenile arrests for homicide
- % of students that report being threatened or injured with a weapon
- % of students bullied at least once a week
- % of students who carried a weapon to school

Ask students to describe how the 2 statistics are correlated. Is there a positive or negative correlation, or is no correlation apparent? Ask students to speculate whether 1 of the trends might be driving the other trend (for example, does the crime rate decrease as a result of fewer juvenile arrests, or do juvenile arrests decrease due to dropping crime rates?) or whether there might be a third factor driving the trend for both of the correlated statistics (such as tougher penalties for crimes or increased conflict counseling). Remind students that a correlation alone never proves causality.
2. Have students compare youth crime rates in the United States to those in other countries. Students can use statistics from Nation Master (www.nationmaster.com/graph/cri_mur_com_by_you_per_cap-murders-committed-youths-per-capita) to create a graph showing the differences among selected countries. Alternatively, have students compare youth crime rates in their state to those in other states. Which countries or states have the highest and lowest crime rates? What are some possible reasons for the differences?

Action Project

Have students visit www.tolerance.org and plan a ‘Mix it Up at Lunch’ day at school where students sit next to people they normally wouldn’t sit next to in order to break boundaries, promote dialogue, and exhibit tolerance. The website contains instructions for conducting activities before, during, and after the actual lunch day.

Additional Resources

- www.takingitglobal.org—TakingITGlobal is a website that connects students from all over the world to global issues and positive action. Visit the Understanding Violence page and learn ways to prevent violence through books, statistics, blogs, links to other websites, and other resources.
- www.peacejam.org—PeaceJam’s goal is to foster a new generation of youth dedicated to creating positive change and peaceful communities through learning about Nobel Peace Laureates.
Investigations

1. Create a double bar graph using the information in the following table. Make sure that years are shown on the $x$-axis.

<table>
<thead>
<tr>
<th>Year</th>
<th>% female students that report being threatened or injured with a weapon</th>
<th>% male students that report being threatened or injured with a weapon</th>
</tr>
</thead>
<tbody>
<tr>
<td>1993</td>
<td>5</td>
<td>18</td>
</tr>
<tr>
<td>1995</td>
<td>5</td>
<td>14</td>
</tr>
<tr>
<td>1997</td>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>1999</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>2001</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>2003</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>2005</td>
<td>3</td>
<td>10</td>
</tr>
</tbody>
</table>

2. Write a sentence explaining the trends observed in your double bar graph.

3. Use the following data to create a line graph, showing the trend in the number of serious violent crimes per 1000 students ages 12-18 in U.S. schools.

<table>
<thead>
<tr>
<th>Year</th>
<th>Violent Crimes per 1000 Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>10</td>
</tr>
<tr>
<td>1993</td>
<td>12</td>
</tr>
<tr>
<td>1994</td>
<td>13</td>
</tr>
<tr>
<td>1995</td>
<td>9</td>
</tr>
<tr>
<td>1996</td>
<td>9</td>
</tr>
<tr>
<td>1997</td>
<td>8</td>
</tr>
<tr>
<td>1998</td>
<td>9</td>
</tr>
<tr>
<td>1999</td>
<td>7</td>
</tr>
<tr>
<td>2000</td>
<td>5</td>
</tr>
<tr>
<td>2001</td>
<td>6</td>
</tr>
<tr>
<td>2003</td>
<td>6</td>
</tr>
<tr>
<td>2005</td>
<td>5</td>
</tr>
</tbody>
</table>

*Note that no data are available for 2002 and 2004.

4. What trend is emerging over time?

5. What are possible causes for the trend you observed?

ANSWERS

1. Percent of Students who Report Being Threatened or Injured with a Weapon

2. Percent of reported threats and injuries with a weapon have decreased over time for both males and females. A higher percent of males report being threatened or injured than females.

3. The data indicate a decrease in student violent crime rates.

4. Possible causes for this trend could be that there is more security in schools, gang numbers have decreased, or students have learned ways to express their anger through other means.
6. Use the following information to create a scatterplot.

<table>
<thead>
<tr>
<th>Year</th>
<th>Homicides on School Grounds during School Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>1993</td>
<td>42</td>
</tr>
<tr>
<td>1994</td>
<td>42</td>
</tr>
<tr>
<td>1995</td>
<td>17</td>
</tr>
<tr>
<td>1996</td>
<td>29</td>
</tr>
<tr>
<td>1997</td>
<td>23</td>
</tr>
<tr>
<td>1998</td>
<td>35</td>
</tr>
<tr>
<td>1999</td>
<td>25</td>
</tr>
<tr>
<td>2000</td>
<td>9</td>
</tr>
<tr>
<td>2001</td>
<td>8</td>
</tr>
<tr>
<td>2002</td>
<td>2</td>
</tr>
<tr>
<td>2003</td>
<td>4</td>
</tr>
<tr>
<td>2004</td>
<td>10</td>
</tr>
<tr>
<td>2005</td>
<td>13</td>
</tr>
<tr>
<td>2006</td>
<td>4</td>
</tr>
<tr>
<td>2007</td>
<td>11</td>
</tr>
</tbody>
</table>

7. Now draw a line of best fit to show how the number of homicides on school grounds during a school day is changing over time. A line of best fit is a straight line (usually drawn using a ruler edge) that best represents the data. There are typically similar numbers of points above and below the line.

8. What pattern does the line of best fit reveal?

9. Based on the 3 graphs you have drawn, what observations can you make about how the number of violent incidents involving students has changed from the early 1990s to the mid 2000s?
Practice with Data & Graphs

A line graph is useful for representing data over a period of time. Multiple categories of data that occur over the same time period can be compared on the same graph, where each line represents a different category.

1. Use the following data\(^1\) to complete the double line graph below. Data on violent crimes per 1000 students has already been graphed. Graph another line for violent crimes per 1000 people.

<table>
<thead>
<tr>
<th>Year</th>
<th>Violent Crimes per 1000 Students</th>
<th>Violent Crimes per 1000 People</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>10</td>
<td>48</td>
</tr>
<tr>
<td>1993</td>
<td>12</td>
<td>49</td>
</tr>
<tr>
<td>1994</td>
<td>13</td>
<td>51</td>
</tr>
<tr>
<td>1995</td>
<td>9</td>
<td>46</td>
</tr>
<tr>
<td>1996</td>
<td>9</td>
<td>42</td>
</tr>
<tr>
<td>1997</td>
<td>8</td>
<td>39</td>
</tr>
<tr>
<td>1998</td>
<td>9</td>
<td>36</td>
</tr>
<tr>
<td>1999</td>
<td>7</td>
<td>32</td>
</tr>
<tr>
<td>2000</td>
<td>5</td>
<td>27</td>
</tr>
<tr>
<td>2001</td>
<td>6</td>
<td>25</td>
</tr>
<tr>
<td>2002</td>
<td>no data available</td>
<td>23</td>
</tr>
<tr>
<td>2003</td>
<td>6</td>
<td>22</td>
</tr>
<tr>
<td>2004</td>
<td>no data available</td>
<td>21</td>
</tr>
<tr>
<td>2005</td>
<td>5</td>
<td>21</td>
</tr>
</tbody>
</table>

2. What trend do you observe for violent crime rates from 1992 to 2005? What are some possible causes of this trend?

3. What is 1 observation you can make about student violence versus overall violence from 1992 to 2005?

4. In 1993, 7 out of 10 local TV stories on violence in California involved youth,\(^2\) yet youth made up only 14% of violent arrests in California that year. Why do you think the news showed so many more stories on youth violence than what was actually occurring?

ANSWERS

1. See graph.

2. Violent crime rates generally decreased among both youth and adult populations. Possible reasons include increased police presence, increased afterschool programs, and increased economic opportunities.

3. Student violent crime rates are much lower. Violent crime rates for the overall population decreased more during the 13-year period.

4. Answers will vary. Possible answers include: youth violence may be more interesting to the public, or parents may be more prone to watch news stories about student violence in order to protect their children.
Students evaluate their personal consumption choices and the factors that influence those choices. Utilizing information in a table, students create an algebraic equation to compare embodied energy in cups made of different materials. After reviewing information related to the environmental impacts and the costs of a number of different cups, students must choose which cup to purchase.
Critical Thinking Questions
• In what ways can we evaluate the sustainability of everyday choices?
• How can personal consumption choices impact the world?

Objectives
• Simplify and compare fractions
• Write fractions as decimals
• Understand how price and production of consumer products impacts their sustainability

Key Concepts
• Converting fractions to decimals
• Algebra
• Consumption

NCTM Standards and Expectations Addressed
Number and Operations: Work flexibly with fractions, decimals, and percents to solve problems
Number and Operations: Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods

Algebra: Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships

Problem Solving: Solve problems that arise in mathematics and in other contexts

Connections: Recognize and apply mathematics in contexts outside of mathematics

Materials/Preparation
Index cards, 1 per student
Student handout: Paper or Plastic?, 1 per student or pair
Teacher master: Paper or Plastic?

Background and Practice
Background reading: Spending Spree
Practice worksheet: Practice with Fractions
**Teacher Instructions**

1. Pass out index cards to students.

2. Ask students to list on their index cards 5 products that they or members of their family bought in the last week.

3. In a think-pair-share activity, have students share the lists from their index cards and answer the following questions:
   - What information is most useful to you in choosing what to buy and use?
   - What information do you think influences people the most when they purchase products?

4. Review the definition of sustainability with students. *Sustainability refers to meeting people's current needs without jeopardizing the ability of people to meet future needs. Three areas included in an assessment of sustainability are society (people), economy, and environment.*

5. Ask students to consider how sustainability (or the ability to meet present and future needs) is related to consumption. *There are a myriad of possible answers to this question, which is intended to get students brainstorming. Consumption patterns may influence sustainability, or vice versa. A consideration of sustainability may influence consumption patterns.*

6. Pass out the handout, *Paper or Plastic?*, to students. Have them work through the investigations either individually or in pairs.

7. If students are not sure how to find the price per cup (for example, the cost of a single Styrofoam cup), have them convert the given fraction into a decimal (i.e., divide the numerator by the denominator).

8. Have students answer the following questions at the end of class, either as a writing activity or as a class discussion.

**Discussion Questions**

1. Which is more important when you make consumer choices: quantity or quality?

2. How does the cost of products affect consumption patterns?

3. What is 1 product that you bought recently that seems like an unsustainable choice? What about the product makes it unsustainable?

4. What are some alternatives to products that you buy or use that are more sustainable than the ones you have used in the past?
Extension Ideas

1. Quantify the fuel saved by recycling different items. Conversions for the amount of gasoline/oil saved by recycling plastic, glass, and aluminum are provided by the Washington State Recycling Foundation. See their flyer, “Energy & Fuel,” at www.wsra.net (click on “News/Resources,” then “Hot Topics”) for a worksheet with conversion factors.

2. The world’s poorest 20% of people consume 1.5% of the world’s products, the middle 60% consume 21.9%, and the richest 20% consume 76.6%. If the population of the world is approximately 6.7 billion people, what is the actual number of people in each consumer category? How does per capita consumption differ among economic groups?

Additional Resources

- www.equalexchange.coop—Equal Exchange is a cooperative organization working to build trade partnerships that are economically just and environmentally sound so that farmers and consumers benefit. You can read about the steps involved in bringing tea, coffee, and chocolate from farm to table.

- www.newdream.org—The “Beyond Consumerism” program from Center for a New American Dream includes research and ideas for the conscious consumer, including product-specific shopping guides.

Action Project

Have students visit www.facingthefuture.org and click on Service Learning, then Fast Facts & Quick Actions. They can click on Consumption, then choose 1 quick action and get started. Encourage them to educate friends and family on how to consider sustainability when making consumption choices.
Investigations

You will use information from the following table to make an informed choice about which kind of cup you would purchase.¹

Table 1. Background Information for Cup Types

<table>
<thead>
<tr>
<th></th>
<th>Cost (price/quantity)</th>
<th>Weight of 1 Cup (in grams, g)</th>
<th>Energy Used per Cup (in megajoules, MJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plastic</td>
<td>$11/1000</td>
<td>59</td>
<td>6.3</td>
</tr>
<tr>
<td>Styrofoam</td>
<td>$25/1000</td>
<td>1.9</td>
<td>0.20</td>
</tr>
<tr>
<td>Paper</td>
<td>$40/500</td>
<td>8.3</td>
<td>0.55</td>
</tr>
<tr>
<td>Ceramic</td>
<td>$20/3</td>
<td>292</td>
<td>14</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>$15/1</td>
<td>378</td>
<td>8.1</td>
</tr>
</tbody>
</table>

1. Use the information from Table 1 to complete the following table.

Table 2. Price and Reusability of Cups

<table>
<thead>
<tr>
<th></th>
<th>What is the price of 1 cup?</th>
<th>Can the cup be reused many times?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plastic</td>
<td>$0.01</td>
<td>No, but might be reused on a limited basis</td>
</tr>
<tr>
<td>Styrofoam</td>
<td>$0.03</td>
<td>No, but might be reused on a limited basis</td>
</tr>
<tr>
<td>Paper</td>
<td>$0.08</td>
<td>No</td>
</tr>
<tr>
<td>Ceramic</td>
<td>$6.67</td>
<td>Yes</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>$15</td>
<td>Yes</td>
</tr>
</tbody>
</table>

2. Based on these prices, how many paper cups could you buy for the price of 1 ceramic mug?

3. Create an algebraic equation that would allow you to determine the number of plastic cups that can be produced using the same amount of energy used to produce 1 ceramic mug.

4. Use mental math to solve the equation you created in the previous problem to determine approximately how many plastic cups could be produced using the same amount of energy that it takes to produce 1 ceramic mug. Report your answer as a whole number.

Answers

1. See table.
2. $6.67/$0.08 \approx 83
3. 6.3x = 14
4. 2
5. How many times would you need to reuse the ceramic mug to use less total energy than getting a new plastic cup each time?

6. What other kinds of information would help you decide which cup you want to purchase?

7. List at least 1 pro and 1 con for purchasing each type of cup.

**Plastic**
- Pro: _________________________________
- Con: _________________________________

**Styrofoam**
- Pro: _________________________________
- Con: _________________________________

**Paper**
- Pro: _________________________________
- Con: _________________________________

**Ceramic**
- Pro: _________________________________
- Con: _________________________________

**Stainless Steel**
- Pro: _________________________________
- Con: _________________________________

8. Which cup would you purchase? Give reasons for your answer.

**Bonus**

People in the United States use 100 billion plastic bags each year, made from an estimated 12 million barrels of oil. If U.S. consumers were to decrease their use of plastic bags by \( \frac{1}{4} \) each year for the next 5 years, how many bags would they use in year 5?

### Answers

5. \( 6.3x > 14 \)
   \[ x > 2.2, \text{ so you would need to use the ceramic mug a minimum of 3 times} \]

6. Possible answers: other resources used for production such as water, greenhouse gases released, toxic chemicals used or released, hazards to workers, whether the materials can be recycled, information about the companies that make the cups

8. Answers will vary.

**Bonus**

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Bags Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100 billion</td>
</tr>
<tr>
<td>1</td>
<td>( \frac{3}{4}(100 \text{ billion}) = 75 \text{ billion} )</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{3}{4}(75 \text{ billion}) = 56.25 \text{ billion} )</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{3}{4}(56.25 \text{ billion}) = 42.1875 \text{ billion} )</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{3}{4}(42.1875 \text{ billion}) = 31.640625 \text{ billion} )</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{3}{4}(31.640625 \text{ billion}) = 23.73046875 \text{ billion} )</td>
</tr>
</tbody>
</table>
**Fractions** represent a part of a whole. For example, imagine dividing a pie into 5 equal pieces. If you eat 2 of those pieces, you have eaten \( \frac{2}{5} \) of the pie.

Consider the following facts:

1. One-third of the planet’s natural resources were consumed over the course of the past 3 decades.
2. 70 garbage cans of waste were produced to make the materials in a single can of garbage discarded in the United States.
3. Of the 80,000 chemicals that are in the materials we buy and use, 15,000 have been tested for safety.
4. Average daily water consumption per person in the United States is 425 liters per day.

1. During the past 30 years, what fraction of the planet’s natural resources was used each year?
2. If you throw out 3 cans of garbage every 2 weeks, how many cans of waste would it take to produce 1 month’s worth of your discarded materials?
3. What fraction of chemicals has been tested for safety concerns? Write your fraction in lowest terms.
4. If U.S. water usage was decreased by \( \frac{1}{5} \), what would be the average water consumption per person per day?

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**ANSWERS**

1. \( \frac{1}{3} \div 30 = \frac{1}{90} \)
2. \( \frac{3 \text{ cans}}{2 \text{ weeks}} = \frac{6 \text{ cans}}{4 \text{ weeks}} \)
   \[ 6 \times 70 = 420 \text{ cans} \]
3. \( \frac{15,000}{80,000} = \frac{3}{16} \)
4. \( 425 \times \frac{1}{3} = 340 \text{ liters} \)
Students begin by analyzing information from a credit card offer. They then work through calculations to analyze financial choices made by 2 young people. The activity connects to national budgeting and debt by investigating the revenues and expenditures of 2 countries.
Objectives
• Work flexibly with fractions and percents
• Understand how interest accrued over time impacts overall costs
• Understand how nations make and spend money
• Evaluate financial choices of individuals and governments

Critical Thinking Questions
• What personal budget choices lead to debt?
• How is national debt connected to a country’s well-being?

Key Concepts
• Fractions
• Percent
• Interest
• Debt

NCTM Standards and Expectations Addressed
Number and Operations: Work flexibly with fractions, decimals, and percents to solve problems
Number and Operations: Compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line
Number and Operations: Develop meaning for integers and represent and compare quantities with them
Number and Operations: Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers
Number and Operations: Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods
Problem Solving: Solve problems that arise in mathematics and in other contexts
Communication: Communicate mathematical thinking coherently and clearly to peers, teachers, and others
Connections: Recognize and apply mathematics in contexts outside of mathematics

Materials/Preparation
A credit card offer, either projected with a document camera or photocopied, 1 per student pair (you can find a variety of credit card offers at www.creditcards.com)
Student handout: Did You Budget for That?, 1 per student pair
Teacher master: Did You Budget for That?
Student handout: National Spending Spree, 1 per student pair (you may want to copy this on the back of the handout, Did You Budget for That?)
Teacher master: National Spending Spree

Background and Practice
Background reading: Something for Nothing?
Practice worksheet: Practice with Percent
Teacher Instructions

1. Show students a credit card offer you received in the mail or that you printed from a website, either projected with a document camera or photocopied for student pairs to analyze.

2. Ask students to work with a partner to review the credit card offer. Together they will come up with at least 2 questions that they would want answered in order to assess whether accepting the credit card offer is a good idea.

3. Have student groups share their questions with the class.

4. The following information may be useful as students bring up questions:
   - **Interest**—Banks and credit card companies do not lend money for free—they charge interest. Interest is the fee you pay in order to borrow money from a bank or other lender.
   - **APR (Annual Percentage Rate)**—APR refers to the minimum interest rate charged. The APR does not include fees, such as a setup fee or an annual fee.
   - **Preapproved**—Banks or other lending agencies may preapprove a person for credit if that person meets a minimum requirement, such as a certain credit score.
   - **Annual fee**—An amount of money that a credit card holder pays once a year for the right to have the card.
   - **Balance transfer**—In many cases, you can transfer the balance from an existing credit card to a new credit card with a lower interest rate.
   - **What happens if you miss a payment?**—Typically you will be charged a late payment fee (around $40) and your interest rate will automatically increase (to around 30%). If you have a credit card issued by a company with a *universal default clause*, missing a payment for any credit card or bill may result in a higher interest rate for your credit card, even if you weren’t late paying that particular credit card bill. These credit card companies periodically check your credit history to review your credit payment history.

5. Distribute 1 *Did You Budget for That?* handout to each student pair. Allow students 20–25 minutes to work through the handout.

6. After most of the class has completed the handout, ask students to share their thoughts on how each of the 2 individuals spent their money. Which budgeting pattern is most similar to their own? Which individual will likely have the most money saved for an emergency?
7. Now ask students how a country or nation is similar to an individual in terms of money. *(Both nations and individuals earn income and spend money. Both can have a surplus of money, and both can be in debt. Both can borrow money.)* Challenge students to brainstorm ways in which a nation makes money *(taxes, exports, etc.)* and ways in which a nation spends money *(e.g. military, social programs, national parks, government employee salaries).*

8. Distribute the handout, *National Spending Spree,* to student pairs. Give students 10-15 minutes to work through the handout. You may need to define *revenue* (money taken in) and *expenditures* (money spent).

9. Conclude the class with a discussion using the following questions.

**Discussion Questions**

1. The average number of credit cards in an American household is almost 13 cards.¹ Why do you think so many people use credit to buy things?

2. Sometimes buying something on credit or obtaining a loan is a good idea, and other times it is not. What are some suggestions you would give to a person who wants to purchase something on credit so that he or she could use credit in the safest manner?

3. When a nation spends more money than it has, it must borrow money. From whom do you think nations borrow money? *(In some cases, nations borrow money from their citizens, such as when the government sells savings bonds. People purchase them today, providing the government with immediate money. The government, in turn, promises to pay the money back at a higher rate of return in the future. In other cases, nations borrow money from other nations or international organizations such as the World Bank or the International Monetary Fund.)*

4. What might be some implications, both positive and negative, of nations borrowing money from other nations?
Extension Ideas

1. Have students review and compare budgets for several nations. Information related to military and education spending can be found in the CIA World Factbook (https://www.cia.gov/library/publications/the-world-factbook). Which nations can sustain their current budgets indefinitely? Which nations must borrow money? What consequences might follow if a country continues to spend more money than it takes in?


3. Have students compare credit card offers (see www.creditcards.com or a similar website) to determine which credit card would be the best “deal” for someone in their family. What are potential benefits of having this credit card? What are potential negative consequences of having this credit card? Why does this credit card appear to be the best choice?

Additional Resources

• www.financial-education-icfe.org—The Institute of Consumer Financial Education website has a section called “Children and Money” that includes financial education resources for children and their families.

• www.webwinder.com/wwhtmbin/java_ccl.html—This Credit Card Minimum Payment Interest Calculator allows you to determine the time and interest required to pay off a credit card if only the minimum required payment is made each month.

Action Project

Start a financial literacy campaign at your school. Have students put together short presentations that teach their peers about credit, debt, interest, and other personal finance issues. Organize a financial education day, when the presentations can be given during lunchtime or during daily announcements. Also, have students create a pamphlet with suggested “Dos” and “Don’ts” of managing and borrowing money. Make pamphlets available in the counselor’s office.
**Scenario 1:** Jason is a senior at West High School. He works at a skateboard shop making $9 an hour. He works about 25 hours each month. He has been saving money so that he can buy a used car when he graduates.

1. If Jason works 25 hours this month and $\frac{1}{4}$ of his paycheck goes to taxes, how much money will he take home?

2. Jason takes his little brother to the movies every week, and he usually eats out at least once a week. In all, he spends about $20 each week and saves the rest of his paycheck after taxes. How much does he save each month?

3. The car Jason wants will cost him $1420. He decides that he will buy it all at once, rather than borrowing the money. For how many months will he need to save to buy it?

4. Jason has been saving money for 12 months, and he plans to work 5 more months at the skateboard store before he graduates. Do you think he has done a good job of budgeting to buy the car? Why or why not?

**Scenario 2:** Jadine is in ninth grade, and she does odd jobs to make money. She mows lawns, rakes leaves, walks pets, washes cars, and babysits for her neighbors. She usually charges $10 an hour for any outdoor work and $5 an hour for babysitting.

5. Jadine wants to buy a Nintendo Wii gaming system. The Wii is $300 plus 9% sales tax. How much does the Wii cost altogether, including tax?

6. Jadine has $109 in a savings account. What fraction of the Wii could her savings pay for?

7. Even though Jadine can’t pay for the Wii right now, she wants to get it now so that she can play it over the weekend. She convinces her mother to put it on their family’s credit card, which charges 18% interest. Jadine tells her mother that she will be able to pay back $25 each month. By paying $25 each month, she will end up paying $367.65 for the Wii. How much more will Jadine pay for the Wii by using the credit card than she would have paid if she bought it with money she had saved up?

8. Approximately how many additional hours of babysitting will Jadine need to do to pay for the Wii if it is charged to the credit card?

9. If she pays $25 each month to the credit card company, how many months will it take her to pay the entire $367.65?

10. Do you think Jadine has done a good job of budgeting to buy the Wii? Why or why not?
National Spending Spree

Country 1
- Total revenues = $2500 billion
- Total expenditures = $2700 billion
- Military spending = $300 billion
- Education spending = $60 billion

Country 2
- Total revenues = $675 billion
- Total expenditures = $660 billion
- Military spending = $55 billion
- Education spending = $81 billion

1. What fraction of Country 1’s total expenditures is made up of military spending? (Write your answer in lowest terms.)

2. Spending on health care for Country 1 amounts to \( \frac{1}{10} \) of military expenditures. How much money does Country 1 spend on health care?

3. Country 1 spends more money than it takes in. How do you think this is possible?

4. What fraction of Country 2’s total expenditures is made up of military spending? (Write your answer in lowest terms.)

5. Which country spends a larger fraction of its expenditures on military spending?

6. What percent of its revenues does each country budget for education?

7. Which country do you think can sustain itself for a longer period of time? Explain why in 1 sentence.

8. If you were the president of a country, would you spend a larger fraction of the budget on military, education, or health care? Why?

9. In your opinion, what are some of the most important uses of tax dollars? Name at least 2.

ANSWERS

1. \( \frac{300 \text{ billion}}{2700 \text{ billion}} = \frac{1}{9} \)

2. \( \frac{1}{10} (300 \text{ billion}) = 30 \text{ billion} \)

3. The country borrows money, just like an individual person would.

4. \( \frac{55 \text{ billion}}{660 \text{ billion}} = \frac{1}{12} \)

5. Country 1: \( \frac{1}{9} > \frac{1}{12} \)

6. Country 1: \( \frac{60}{2500} \times 100 = 2.4\% \)
   
   Country 2: \( \frac{81}{675} \times 100 = 12\% \)

7. Country 2 can sustain itself because it does not spend more money than it has.

8. Answers will vary.

9. Possible answers: maintaining and building roads, education, military defense, medical research, environmental protection, social security, health care.
### Practice with Percent

**Percent** is a ratio that compares a number to 100. For example, \( \frac{25}{100} \) is 25%.

To **write a fraction as a percent**, divide the numerator by the denominator and multiply the quotient by 100. For example, the fraction \( \frac{4}{5} \) can be rewritten as 80%:

\[
\frac{4}{5} = 0.80 \times 100 = 80\%
\]

To **turn a percent into a fraction**, remove the percent sign (%). The resulting number is the numerator, and 100 is the denominator. For example, 50% can be rewritten as \( \frac{1}{2} \):

\[
50\% = \frac{50}{100} = \frac{1}{2}
\]

1. Charles makes $7.25 per hour. He works after school Monday–Friday from 4–6 PM and on Saturday and Sunday from 10–3 PM. He puts 10% of every paycheck toward savings for college.

   a. How much money does Charles earn each week?

   b. How much does Charles save each week for college?

   c. Based on your last answer, calculate how much Charles saves in 1 year for college.

   d. If Charles is planning to save the rest of his earnings each week so that he can buy a new laptop for $1500, how many weeks will it take him to save enough money?

2. If 17 million U.S. students are in college and 75% own at least 1 credit card,\(^1\) how many students around the country own a credit card?

3. One in 20 households in America owes $8,000 or more in credit card debt.\(^2\) What is the percentage of households that owe more than $8,000 in credit card debt?

4. Country Z’s expenditures are approximately 30 billion dollars.

   a. If country Z spends 3.5% of its expenditures on education, how much has it spent?

   b. Country Z spends 9 billion dollars on universal health care. Approximately what percent of country Z’s expenditures does it spend on health care?

### ANSWERS

1. a. $7.25/hour \times 20 \text{ hours} = $145
   
   b. $145 \times 0.10 = $14.50
   
   c. $14.50 \times 52 \text{ weeks} = $754
   
   d. Charles saves $130.50 each week. ($145 - $14.50)
      
      It will take him 12 weeks. ($1500 \div $130.50 \approx 11.49$)

2. 17 million \times 0.75 = 12.75 million, or 12,750,000 students

3. \( \frac{1}{20} = 5\% \)

4. a. 30 billion \times 0.035 = 1.05 billion, or $1,050,000,000
   
   b. \( \frac{9}{30} = 30\% \)
Students compare life expectancy among several countries and discuss possible explanations for observed differences. Life expectancy is correlated with other statistics for multiple countries. During a class discussion, students analyze the usefulness of life expectancy as an indicator of a country’s well-being.
Critical Thinking Questions
• Why do people in some countries tend to live longer than people in other countries?
• What factors contribute to long life expectancy?

Objectives
• Calculate percent change in life expectancies
• Use a scatterplot and lines of best fit to evaluate data
• Correlate indicators of well-being with life expectancy for a country
• Understand the concept of probability
• Investigate global trends in health and life expectancy

Key Concepts
• Correlation
• Percent change
• Probability
• Global health and well-being
• Life expectancy

NCTM Standards and Expectations
Number and Operations: Work flexibly with fractions, decimals, and percents to solve problems
Number and Operations: Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods

Number and Operations: Develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results

Algebra: Use graphs to analyze the nature of changes in quantities in linear relationships

Data Analysis and Probability: Select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots

Data Analysis and Probability: Make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit

Problem Solving: Solve problems that arise in mathematics and in other contexts

Reasoning and Proof: Make and investigate mathematical conjectures

Communication: Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Connections: Recognize and apply mathematics in contexts outside of mathematics

Representation: Use representations to model and interpret physical, social, and mathematical phenomena
Materials/Preparation

Cards: *Life Expectancy Country Cards*, 1 per student

Clear a large space in the room so that students can stand in a line at the back of the class and take several steps forward

Teacher master: *Take a Step for Life Readings*

Student handout: *Live Long and Prosper*, 1 per student

Teacher master: *Live Long and Prosper*

Graph paper, 1 page per student

Ruler or other straightedge device, 1 per student

Background and Practice

Background reading: *A Global Checkup*

Practice worksheet: *Practice with Rates, Ratios, & Proportions*

Teacher Instructions

1. Ask students to consider whether they want to live a long time and why. Allow a couple of volunteers to share their answers.

2. Ask the class what kinds of things might help them to live long lives.

3. Ask if anyone can define “life expectancy.” *Life expectancy is the average number of years for a given population that an individual born today would be expected to live if current mortality rates do not change. It is often used as an indicator of the well-being of a country.*


5. Stand in front of the class and tell them: “You have all received different cards with life expectancy information for specific countries. While an individual can influence the length of his or her lifespan through behavior and activities, a country’s life expectancy gives you an idea of the average number of years that any individual born in that country today will be expected to live.”

6. Read aloud to the class from the teacher master, *Take a Step for Life Readings.* Read the paragraphs in the order given. Pause frequently. Each life expectancy group will be directed to step forward before hearing the description of their group.

7. While students are still standing, ask the class why they think the countries in the front line (that is, those with life expectancies greater than 75 years) have such long life expectancies. Why might life expectancies be so low for the countries in the back line?

8. Asks students to return to their seats.

9. Pass out the handout, *Live Long and Prosper.* Have students work on the handout either individually or in small groups.
10. To calculate percent change in life expectancy (LE), use the following equation: 
\[ \% \text{ change} = \left( \frac{X_2 - X_1}{X_1} \right) \times 100, \]
where \( X_2 \) is LE in 2000 and \( X_1 \) is LE in 1960.

In other words, the percent change for Cambodia, with a 2000 LE of 53.81 and a 1960 LE of 42.59 would be calculated thus:
\[ \left( \frac{53.81 - 42.59}{42.59} \right) \times 100 = 26.34\% \text{ increase} \]

11. While students are working with correlations, check for their understanding of what a correlation is. *Whereas correlation does imply a relationship, it does not imply causality. We cannot be sure whether a) high life expectancy leads to high GNI per capita, b) high GNI per capita leads to high life expectancy, or c) high life expectancy and GNI per capita are the result of another factor.*

12. If you want to graph the data using Excel, you can insert a trendline on your scatterplot to further investigate the trend. (Go to the Chart menu, and choose Add trendline.) Excel will also calculate \( R^2 \) values, indicating the strength of the correlation. (After choosing Add trendline, choose Display equation on chart and Display R-squared value on chart.)

13. Note: The last problem on the student handout asks students about probability. While there is no way to tell if an individual will live until a minimum age, there are factors that can increase the probability of someone living longer. For example, people who have access to good health care, people who don’t smoke, and people who exercise are individually likely to live longer than people without those characteristics.

**Discussion Questions**

1. Do you think life expectancy is a good way to measure the health of people in a country? Why or why not?

2. What are some other ways you can tell how well people in a country are doing? (e.g., literacy rates, proportion of citizens in jail, number of refugees, per capita income)

3. Do you think that worldwide life expectancy will continue to increase? Why or why not?

4. Probability is a term used to describe how likely something is to happen. How might probability be related to life expectancy? (*Life expectancy is similar to probability in that it tells you the likelihood of something happening—in this case the likelihood of a baby born today surviving to a certain age.*)
Extension Ideas

1. Investigate the life expectancy of other countries or other kinds of information that might be correlated with life expectancy data (for example, average years of educational attainment, percentage of population in rural areas, and percentage of women who receive prenatal care). Use scatterplots and lines of best fit to determine whether certain factors correlate with life expectancy.

2. Some public health researchers believe that income inequality (rich-poor gap) is a major indicator of poor health. Why might this be the case? Can you find data to support or refute this claim?

Additional Resources

- [www.who.int](http://www.who.int)—The World Health Organization (WHO) has up-to-date information and statistics about the world’s most pressing health issues.
- [www.unicef.org](http://www.unicef.org)—UNICEF focuses on improving the health and well-being of children around the world. UNICEF publishes annual statistics related to women’s and children’s health in their *State of the World’s Children* reports.

Action Project

Have students become health ambassadors and teach other students about good health habits. They can learn how to prevent injuries and poisonings from Safe Kids Worldwide ([www.safekids.org](http://www.safekids.org)). Visit your state’s Department of Health website and the Centers for Disease Control (CDC) ([www.cdc.gov](http://www.cdc.gov)) for additional health and safety topics. Spread the news about ways to lead a healthy life through posters, public service announcements, articles for the school newspaper, or pamphlets.
### Life Expectancy Country Cards

<table>
<thead>
<tr>
<th>Country</th>
<th>Region</th>
<th>Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan</td>
<td>Asia</td>
<td>82.1</td>
</tr>
<tr>
<td>Canada</td>
<td>North America</td>
<td>81.2</td>
</tr>
<tr>
<td>Norway</td>
<td>Europe</td>
<td>79.8</td>
</tr>
<tr>
<td>United States</td>
<td>North America</td>
<td>78.1</td>
</tr>
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<td>Costa Rica</td>
<td>Central America</td>
<td>77.4</td>
</tr>
<tr>
<td>Cuba</td>
<td>Caribbean</td>
<td>77.3</td>
</tr>
<tr>
<td>Mexico</td>
<td>North America</td>
<td>75.8</td>
</tr>
<tr>
<td>Sri Lanka</td>
<td>Asia</td>
<td>75.0</td>
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<td>Jamaica</td>
<td>Caribbean</td>
<td>73.6</td>
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<tr>
<td>Venezuela</td>
<td>South America</td>
<td>73.5</td>
</tr>
<tr>
<td>China</td>
<td>Asia</td>
<td>73.2</td>
</tr>
<tr>
<td>Turkey</td>
<td>Middle East</td>
<td>73.1</td>
</tr>
<tr>
<td>Thailand</td>
<td>Asia</td>
<td>72.8</td>
</tr>
<tr>
<td>Uzbekistan</td>
<td>Asia</td>
<td>71.3</td>
</tr>
<tr>
<td>Nicaragua</td>
<td>Central America</td>
<td>71.2</td>
</tr>
<tr>
<td>Peru</td>
<td>South America</td>
<td>70.4</td>
</tr>
<tr>
<td>Ukraine</td>
<td>Eastern Europe</td>
<td>68.1</td>
</tr>
<tr>
<td>Papua New Guinea</td>
<td>Pacific Ocean</td>
<td>66.0</td>
</tr>
<tr>
<td>Russia</td>
<td>Eastern Europe/Asia</td>
<td>65.9</td>
</tr>
<tr>
<td>Yemen</td>
<td>Middle East</td>
<td>62.9</td>
</tr>
<tr>
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<td>Asia</td>
<td>61.7</td>
</tr>
<tr>
<td>Nepal</td>
<td>Asia</td>
<td>60.9</td>
</tr>
<tr>
<td>Haiti</td>
<td>Caribbean</td>
<td>57.6</td>
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<tr>
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<td>Africa</td>
<td>56.6</td>
</tr>
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<td>Cameroon</td>
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<td>Rwanda</td>
<td>Africa</td>
<td>49.8</td>
</tr>
<tr>
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<td>Africa</td>
<td>38.6</td>
</tr>
<tr>
<td>Swaziland</td>
<td>Africa</td>
<td>32.0</td>
</tr>
</tbody>
</table>
Take a Step for Life Readings

**The Shortest Life Spans**
Those of you with country cards with life expectancies of less than 50 years, please take 1 step forward. You have the shortest life expectancies in the world. Approximately 10% of the world’s population lives in countries with life expectancies below 50 years of age. Almost all of these countries are located in sub-Saharan Africa.

**The Middle Life Spans**
Those of you with country cards with life expectancies of 50 to 75 years, please take 2 steps forward. The majority of the world’s population (about 70%) lives in countries that have life expectancies in this range. These countries are located mostly in South America, North Africa, Eastern Europe, Asia, and the South Pacific.

**The Longest Life Spans**
Those of you with country cards with life expectancies of more than 75 years, please take 3 steps forward. You have the longest life expectancies in the world. Approximately 20% of the world’s population lives in countries with life expectancies longer than 75 years of age. These countries include those in North America and Europe, a few in South America and Northern Africa, and the nations of Australia, Japan, Cuba, and New Zealand.
**Investigations**

1. **Percent change** gives you an idea of how much an amount has increased or decreased. For each country in the following table, calculate the percent change in life expectancy from 1960 to 2000.

2. Check for validity of answers. If life expectancy decreased from 1960 to 2000, percent change in life expectancy should be a negative number (percent decrease). If life expectancy increased, percent change should be positive (percent increase).

3. What does percent change in life expectancy tell you?

4. Which countries experienced a percent decrease, or a negative percent change, in life expectancy?

5. Give 2 possible reasons why life expectancy might decrease in a given country.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cambodia</td>
<td>2920</td>
<td>84</td>
<td>0.6</td>
<td>42.59</td>
<td>53.81</td>
<td>26%</td>
</tr>
<tr>
<td>Canada</td>
<td>34,610</td>
<td>2199</td>
<td>0.3</td>
<td>71.11</td>
<td>78.92</td>
<td>11%</td>
</tr>
<tr>
<td>Chad</td>
<td>1230</td>
<td>25</td>
<td>3.5</td>
<td>34.86</td>
<td>48.47</td>
<td>39%</td>
</tr>
<tr>
<td>China</td>
<td>7730</td>
<td>104</td>
<td>0.1</td>
<td>36.31</td>
<td>70.25</td>
<td>93%</td>
</tr>
<tr>
<td>Jamaica</td>
<td>4030</td>
<td>162</td>
<td>1.5</td>
<td>64.37</td>
<td>75.34</td>
<td>17%</td>
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<tr>
<td>Kenya</td>
<td>1300</td>
<td>79</td>
<td>6.1</td>
<td>44.95</td>
<td>46.97</td>
<td>4%</td>
</tr>
<tr>
<td>Mexico</td>
<td>11,330</td>
<td>354</td>
<td>0.3</td>
<td>57.33</td>
<td>72.96</td>
<td>27%</td>
</tr>
<tr>
<td>Nepal</td>
<td>1630</td>
<td>51</td>
<td>0.5</td>
<td>38.51</td>
<td>58.85</td>
<td>53%</td>
</tr>
<tr>
<td>Peru</td>
<td>6070</td>
<td>246</td>
<td>0.6</td>
<td>47.97</td>
<td>69.31</td>
<td>44%</td>
</tr>
<tr>
<td>Sri Lanka</td>
<td>5000</td>
<td>92</td>
<td>&lt;0.1</td>
<td>60.14</td>
<td>73.14</td>
<td>22%</td>
</tr>
<tr>
<td>Swaziland</td>
<td>5170</td>
<td>159</td>
<td>25.9</td>
<td>40.39</td>
<td>45.62</td>
<td>13%</td>
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<tr>
<td>Thailand</td>
<td>9140</td>
<td>321</td>
<td>1.4</td>
<td>52.64</td>
<td>68.81</td>
<td>31%</td>
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<td>Ukraine</td>
<td>7510</td>
<td>219</td>
<td>1.4</td>
<td>69.32</td>
<td>68.28</td>
<td>−2%</td>
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<tr>
<td>United States</td>
<td>44,260</td>
<td>3829</td>
<td>0.6</td>
<td>69.77</td>
<td>77.06</td>
<td>10%</td>
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<td>Venezuela</td>
<td>7440</td>
<td>241</td>
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<td>59.82</td>
<td>73.34</td>
<td>23%</td>
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<tr>
<td>Zambia</td>
<td>1000</td>
<td>39</td>
<td>17.0</td>
<td>41.78</td>
<td>37.97</td>
<td>−9%</td>
</tr>
</tbody>
</table>

**ANSWERS**

1. See table.
2. Check for validity of answers. If life expectancy decreased from 1960 to 2000, percent change in life expectancy should be a negative number (percent decrease). If life expectancy increased, percent change should be positive (percent increase).
3. The degree to which a population's life expectancy has changed (increased or decreased)
4. Ukraine and Zambia
5. Famine, disease, war, devastating natural disaster
6. Investigate 1 of the other statistics listed (GNI per capita, Health Expenditure per capita, or Percent of Population Ages 15-49 with HIV/AIDS) to see if there is a correlation between that statistic and life expectancy in the year 2000. To determine whether a correlation exists, create a scatterplot. For each country, designate your chosen variable as $x$, and Life Expectancy as $y$. Thus, each country will have one ordered pair $(x, y)$ for a total of 16 points on the graph.

7. For the scatterplot you created, would it matter if life expectancy was on the $y$-axis rather than the $x$-axis?

8. Write 1 sentence to describe the relationship between the 2 variables in your scatterplot.

9. Draw a line of best fit on your scatterplot.

10. Suppose a child born in Ethiopia in 1980 had a 70% probability of living until the age of 40. Does this mean the child will definitely live until at least the age of 28?

**Bonus**

Especially in wealthier countries, obesity can lower life expectancy. Obesity is related to health problems such as diabetes, high blood pressure, and poor sleep. In the United States, 37% of adult males and 42% of adult females are obese. If the population of the United States is approximately 300 million, and 50% of the population is male, what is the number of men who are obese?

% Population with HIV/AIDS vs Life Expectancy

7. No, the relationship would remain the same.

8. Health expenditure per person is positively correlated with life expectancy. GNI per capita is positively correlated with life expectancy. Percentage of population ages 15-49 with HIV/AIDS is negatively correlated with life expectancy.

9. See scatterplots.

10. No, because probability is simply the average ‘chance’ that he or she will live until age 40. It means that 30% of children born in Ethiopia in 1980 will likely not live until the age of 40, and 70% of them are likely to live until at least 40. Probability describes a ‘likelihood’ that something will happen, but it is not a certainty.

**Bonus**

Approximately 55.5 million males in the U.S. are obese.
A **rate** represents a relationship between 2 quantities, or numbers, that have different units of measure.

Example: 8 miles per hour (8 miles, 1 hour)

A **ratio** represents a relationship between 2 quantities, or numbers, that have the same units of measure.

Example: 1 person out of 100 (1 person, 100 persons)

1. Label each of the following scenarios as a rate or ratio.

   a. _____ Gross National Income (measured in dollars) per person

   b. _____ Number of people living with HIV out of 100 people

   c. _____ Dollars spent on books versus dollars spent on food

   d. _____ Three meals/day

2. Write a **ratio**, in simplest terms, to represent the following scenario:

   In the United States, 48 children out of 300 children are obese (weigh over 20% more than their ideal body weight).

3. Write a **rate** to represent the following scenario:

   Each year in the United States, individuals with diabetes spend an average of $12,000 on health care.

A **proportion** is an equation that shows how 2 ratios are equal.

Example: \( \frac{1}{5} = \frac{2}{10} \)

To solve a proportion for an unknown variable \( x \), use inverse operations or multiply cross products.

<table>
<thead>
<tr>
<th>Inverse Operations</th>
<th>Multiply Cross Products</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{x}{4} = \frac{3}{12} )</td>
<td>( \frac{x}{4} = \frac{3}{12} )</td>
</tr>
<tr>
<td>( 4(\frac{x}{4}) = 4(\frac{3}{12}) )</td>
<td>( 3 \cdot 4 = x \cdot 12 )</td>
</tr>
<tr>
<td>( x = \frac{12}{12} )</td>
<td>( 12 = 12x )</td>
</tr>
<tr>
<td>( x = 1 )</td>
<td>( \frac{12}{12} = \frac{12x}{12} )</td>
</tr>
</tbody>
</table>

4. Set up and solve the following **proportion**:

   If 85 babies out of 100 births survive to adulthood, how many babies out of 1500 births will survive to adulthood?

   \[ \frac{85}{100} = \frac{x}{1500} \]
   
   \[ x = 1275 \]
Students create connections between individual actions and climate change. They work with algebraic inequalities to consider how many and what types of activities can be performed while staying within a limited carbon budget.
Critical Thinking Questions
• What are some personal choices that can contribute to or lessen the impacts of climate change?
• How do the choices of some affect everyone regarding climate change?

Objectives
• Graph an inequality on a number line
• Solve inequalities using multiplication and division
• Work with decimals
• Recognize that certain activities release CO₂ and other greenhouse gases
• Understand how CO₂ is related to global climate change
• Investigate actions to reduce our contributions to climate change

Key Concepts
• Solving inequalities
• Graphing inequalities
• Greenhouse gas emissions
• Carbon footprint
• Climate change

NCTM Standards and Expectations
Number and Operations: Work flexibly with fractions, decimals, and percents to solve problems
Number and Operations: Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods
Algebra: Develop an initial conceptual understanding of different uses of variables
Algebra: Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships
Problem Solving: Solve problems that arise in mathematics and in other contexts
Communication: Communicate mathematical thinking coherently and clearly to peers, teachers, and others
Connections: Recognize and apply mathematics in contexts outside of mathematics
Representation: Use representations to model and interpret physical, social, and mathematical phenomena

Materials/Preparation
Student handout: Carbon Footprint Survey, 1 per student
Student handout: Budgeting for Climate Change, 1 per student or pair
Teacher master: Budgeting for Climate Change

Background and Practice
Background reading: Carbon, Carbon Everywhere!
Practice worksheet: Practice with Inequalities
**Teacher Instructions**

1. Pass out 1 *Carbon Footprint Survey* to each student. Have students administer the survey to each other in pairs. Give students a couple of minutes to interview each other and record their answers.

2. In a think-pair-share activity, ask students what they think a carbon footprint might be (based on the survey they just took). Has anyone heard of carbon (or carbon dioxide)?

3. Tell them that each of us has a **carbon footprint**, which is a measure of all the activities we do that release, or cause **emissions**, of carbon dioxide.

4. Ask students what they have heard about carbon dioxide. If nobody mentions the connection to climate change, ask the class what comes to their minds when they hear the words “climate change.”

5. If students are not at all familiar with climate change, you may want to introduce the term **greenhouse gases**. Greenhouse gases such as carbon dioxide and methane are released during the combustion of fossil fuels such as coal and petroleum. We often use these fuels for transportation and electricity. Greenhouse gases accumulate in Earth’s atmosphere and act like a blanket, trapping energy from the sun near Earth’s surface, which results in warmer average temperatures.

6. Students may ask what is wrong with warmer average temperatures. Brainstorm what might happen if there were changes in temperature. **Severe floods, changes in weather patterns, melting glaciers, a decrease in wildlife, and possible health-related issues.**

7. Go through the survey 1 question at a time, asking students how they think each choice directly connects to climate change. **Any transportation and electricity choices that involve fossil fuels such as coal and oil contribute to climate change. Meat production has a larger impact on Earth’s climate than vegetable production. Recycling generally is more energy efficient than manufacturing from raw materials. Turning off appliances, electronics, and lights when not in use is a good way to reduce our carbon footprint from electricity.**

8. Ask students to brainstorm some other activities they do everyday that they think are related to climate change. **Anything that requires energy from fossil fuels releases greenhouse gases. Cutting down trees and other plants results in more carbon dioxide in the atmosphere. Livestock production is a source of methane.**

9. Now ask them to brainstorm activities they do that they do not think contribute to climate change, or activities they do to help reduce the effects of climate change. On the Carbon Footprint Survey, have
each student write 1 way in which he or she can help reduce the impacts of climate change.

10. Pass out the handout, *Budgeting for Climate Change*. Allow students to work with a partner or independently. Tell students that they should carry out their answers to 2 decimal places.

11. After students have worked through the problems on the worksheet, lead a class discussion using the following questions.

**Discussion Questions**

1. Do you think setting a maximum amount of carbon emissions (for a person, a country, or a business) is a good way to reduce the impacts of climate change? Are there other climate change solutions that you think would work better?

2. Do you think it will be difficult for people in the United States to reduce their daily carbon emissions? Why or why not?

3. Do you think all people should be required to reduce their emissions equally? Explain your reasoning.

4. What actions do you think would be the easiest for us to reduce our greenhouse gas emissions? What actions do you think could reduce our carbon footprint the most?

**Extension Ideas**

- Have students research the carbon emissions savings that would result over 1 year for a single lifestyle change (for example, keeping the house cooler in the winter, or biking to school once a week). How many fewer emissions would result if everyone in the United States made the same lifestyle change?

- Have students research cost-effective ways of reducing greenhouse gas emissions (e.g., compact fluorescent light bulbs, sealing cracks around windows and doors, unplugging appliances when not in use). Give students a “budget” of $50 and challenge them to find the most effective ways to reduce CO₂ emissions within that budget. How many pounds of CO₂ can their $50 prevent from entering Earth’s atmosphere?
Additional Resources

• [www.zerofootprintkids.com](http://www.zerofootprintkids.com)—The Zero-footprint KidsCalculator allows students to measure their impact on climate change. Results are given in 4 units of measurement: carbon, land, water, and total footprint. Individuals can compare their results to average results for a variety of countries.

• [www.nature.org/greenliving/carbon_calculator](http://www.nature.org/greenliving/carbon_calculator)—The Nature Conservancy’s carbon calculator starts with an average carbon footprint and subtracts CO₂ emissions for actions that reduce emissions.

• [www.facingthefuture.org](http://www.facingthefuture.org)—Download the Facing the Future curriculum unit, Climate Change: Connections and Solutions. The 2-week interdisciplinary unit encourages students to think critically about climate change and to collaborate to devise solutions. Students learn about climate change within a systems framework, examining interconnections among environmental, social, and economic issues.

Action Project

Have students interview an adult using the questions from an online carbon calculator such as the Zerofootprint KidsCalculator ([www.zerofootprintkids.com](http://www.zerofootprintkids.com)) or the EPA greenhouse gas emissions calculator ([www.epa.gov/climatechange/emissions/ind_calculator.html](http://www.epa.gov/climatechange/emissions/ind_calculator.html)). After the adult has answered the questions, the student can enter the information into the calculator to find the adult’s carbon footprint. Then, have students come up with ideas for specific ways to reduce their interviewee’s carbon footprint. Encourage students to present their footprint results and ideas to their interviewees.
## Carbon Footprint Survey

1. How do you get to school?
2. Do you fly on a plane often?
3. Do you eat mostly vegetables or mostly meat?
4. Do you recycle?
5. Do you turn off your computer when you’re not using it?
6. Do you turn off lights when you leave a room?
Investigations

1. One way that we contribute to climate change is by doing things that release CO₂ into the air. The Kyoto Protocol is an agreement among nations that sets a target for maximum CO₂ emissions for each nation. The U.S. emissions target is 24.80 pounds per person per day.¹

Graph the following inequality on a number line: \( x \leq 24.80 \)

2. Using information from the following table, write an inequality that would allow you to calculate the number of miles you can drive in an average car and remain below 24.80 pounds of CO₂. Use the variable \( m \) to represent number of miles.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Emissions (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drive 1 mile in an average car</td>
<td>1.10 *</td>
</tr>
<tr>
<td>Drive 1 mile in a SUV</td>
<td>1.67 *</td>
</tr>
<tr>
<td>Eat 1 cheeseburger</td>
<td>6.78 **</td>
</tr>
<tr>
<td>Eat 1 PB&amp;J sandwich</td>
<td>2.50 ***</td>
</tr>
<tr>
<td>Read and throw away a newspaper</td>
<td>0.50 ****</td>
</tr>
<tr>
<td>Use and throw away an aluminum can</td>
<td>0.45 ****</td>
</tr>
<tr>
<td>Take hot shower for 1 minute</td>
<td>1.10 ****</td>
</tr>
<tr>
<td>Leave appliances plugged in when not in use</td>
<td>0.27 *****</td>
</tr>
</tbody>
</table>

3. Now solve the inequality you wrote for question #2 to determine the number of miles you could drive in a day without exceeding 24.80 pounds of CO₂ emissions.

4. Do you think it would be easy or difficult for most people to drive less than that number of miles on a daily basis? Why or why not?

5. How could people travel without using cars?
6. The following equation allows us to determine how many miles a person can drive in a day and how many 5-minute showers a person can take and still remain below 24.8 pounds of CO₂ emissions.

\[ 5.5s + 1.1m \leq 24.8 \]

If you take 2 showers (that is, \( s = 2 \)), what is the maximum number of miles you can drive?

7. Approximately how many PB&J sandwiches could you eat before you created more emissions than 1 cheeseburger?

8. If you throw away newspapers, throw away aluminum cans, and leave appliances plugged in when you are not using them, how many emissions would be produced from those activities in a single day?

9. Approximately what fraction of the maximum CO₂ emissions (24.8) is represented by the total emissions from question #8?

   Hint: Round each number to the nearest whole number.

Bonus

The annual carbon footprint of the average person living in the United States—20.4 tons of CO₂—is around 2,000 times that of someone living in the African nation of Chad.²

What is the annual carbon footprint (in pounds of CO₂) of the average person living in Chad?

What is the minimum number of individual carbon footprints in Chad that would be required to exceed the average CO₂ emissions of 1 person in the U.S.?

---

**ANSWERS**

6. \( 5.5(2) + 1.1m \leq 24.8 \)
   \[ 11 + 1.1m \leq 24.8 \]
   \[ 1.1m \leq 13.8 \]
   \[ m \leq 12.54 \text{ miles} \]

7. \( 6.78 \div 2.50 \approx 3 \)

8. \( 0.50 + 0.45 + 0.27 = 1.22 \text{ lbs} \)

9. \( \frac{1}{25} \)

**Bonus**

1 ton = 2,000 pounds

The U.S. footprint (20.4 tons, or 40,800 pounds) is 2,000 times greater than the Chad footprint.

\[ 40,800 \text{ pounds} \div 2000 = 20.4 \text{ pounds} \]

If 2000 Chad footprints = 1 U.S. footprint, then 2001 Chad footprints > 1 U.S. footprint.
There are 4 inequality symbols to remember:

- $x < 5$ means $x$ is a number less than 5
- $x \leq 5$ means $x$ is a number less than or equal to 5
- $x > 5$ means $x$ is a number greater than 5
- $x \geq 5$ means $x$ is a number greater than or equal to 5

Solving an inequality using addition or subtraction is similar to solving an equation. Any operation (addition or subtraction) that you do on one side of the inequality must also be done on the other side.

\[
x + 5 < 20
\]
\[
-5 -5
\]
\[
x < 15
\]

If you multiply or divide both sides of an inequality by a positive number, the inequality sign remains unchanged.

\[
3x \geq 27
\]
\[
\frac{3}{3} \geq \frac{27}{3}
\]
\[
x \geq 9
\]

When you multiply or divide both sides of an inequality by a negative number, the inequality symbol reverses.

\[
-2x + 5 > 25
\]
\[
-5 -5
\]
\[
-2x > 20
\]
\[
-2 \quad -2
\]
\[
x < -10
\]

1. Suppose a person uses nearly 3,000 more kilowatt-hours of electricity in a year than the average American household. If the average American household uses 10,656 kilowatt-hours of electricity, solve the following inequality to find that person’s electricity usage:

\[
x - 3,000 < 10,656
\]

2. Write a sentence explaining your answer for problem #1.

3. Suppose a city institutes a water restriction to conserve fresh water resources during hot summer months. The city will allow each resident to consume no more than 85 gallons of water per person per day. Write an inequality to represent this scenario.

4. \((-\frac{1}{3})x > 30\)

5. \(-3 + 5x > 25\)

6. \(25 \leq -x\)

ANSWERS

1. \(x < 13,656 \text{ kWh}\)

2. The person uses less than 13,656 kWh per year.

3. \(x \leq 85\)

4. \(x < -90\)

5. \(x > \frac{3}{5} \text{ or 5.6}\)

6. \(-25 \geq x\)
Students reflect on ways in which they use energy for everyday activities and the fuels used to power those activities. Students use geometric and trigonometric formulas to calculate unknown angles and side lengths of a triangle formed by a solar panel that is tilted to optimize incoming solar radiation. Implications of utilizing renewable energy sources, including solar power, are considered in light of climate change and energy security.
Critical Thinking Questions
• How is energy use connected to other global issues?
• How can solar energy collection be optimized at different latitudes?

Objectives
• Determine missing angle measurements of a triangle
• Classify angles and triangles as acute, obtuse, or right
• Identify supplementary angles
• Use the Pythagorean theorem to calculate the length of a missing side of a right triangle
• Use trigonometric ratios to determine the length of a missing side of a triangle
• Explore solar power as a form of renewable energy

Key Concepts
• Classifying triangles and angles
• Measuring angles
• Supplementary angles
• Pythagorean theorem
• Trigonometric ratios
• Renewable and nonrenewable energy sources

NCTM Standards and Expectations Addressed
Geometry: Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects

Geometry: Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life
Measurement: Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision
Problem Solving: Solve problems that arise in mathematics and in other contexts
Communication: Communicate mathematical thinking coherently and clearly to peers, teachers, and others
Connections: Recognize and apply mathematics in contexts outside of mathematics
Representation: Use representations to model and interpret physical, social, and mathematical phenomena

Materials/Preparation
Student handout: Quick Quiz, 1 per group of 4 students
Student handout: What’s Your Angle?, 1 per student
Teacher master: What’s Your Angle? (Optional) A world map or atlas

Background and Practice
Background reading: Energy and Sustainability
Practice worksheet: Practice with Angles
Teacher Instructions

1. Divide the class into groups of 4. Pass out 1 Quick Quiz to each group.
2. Ask students what it means for something to be renewable. It can be used over and over again.
3. Give student groups a few minutes to read, discuss, and answer the questions on the Quick Quiz.
4. Review answers to the quiz. 1. b—ancient plants; 2. a—7%; 3. c—petroleum oil; 4. f—hair dryer
5. Ask if anyone knows what fuels are used to create electricity. Much of our electricity comes from coal, a nonrenewable fuel (it cannot be replaced as quickly as we consume it). Many places also derive electricity from renewable fuels, such as solar energy, wind energy, and hydroelectric power.
6. Ask the class if they know how nonrenewable fuels (including fossil fuels such as coal, oil, and natural gas) are connected to climate change. Greenhouse gases such as carbon dioxide are released when we burn these fuels. Those greenhouse gases accumulate in Earth’s atmosphere acting like a blanket. This traps energy from the sun near Earth’s surface and results in warmer temperatures on Earth. Solar energy is 1 renewable source of power that does not contribute to climate change as it is used. Energy from the sun is transformed into electricity by photovoltaic cells, commonly called solar panels.
7. Have students complete the handout, What’s Your Angle?
8. If students are not familiar with the concept of latitude, review this concept. Latitude refers to the angular distance between the equator and a particular location in the world. It is often measured in degrees north or south of the equator. Some maps and globes show latitudes as horizontal lines that run parallel to the equator.
9. For question #2, you may want to have students find the latitude of your city and calculate the optimal angle of tilt based on this angle. Students could also study the given latitudes on a map or atlas. Ask them to draw a conclusion about why the optimum angles of tilt are different based on the given locations.
10. For question #9, make sure they use the formula: tangent of angle = opposite angle/adjacent angle
11. Encourage students to think critically about applications of solar power by discussing answers to the following questions.
Discussion Questions

1. Have you seen a photovoltaic cell before? Where? (e.g., calculators, watches, solar panels)

2. Do you think solar power would be a practical source of energy where you live? Why or why not?

3. What are possible advantages to using solar energy? What are possible disadvantages? (Advantages may include: solar energy is renewable, it doesn't pollute the air, and photovoltaic cells are easy to install. Disadvantages may include: solar energy is expensive and cannot be used at night.)

4. If you want to encourage use of a particular energy source in your community, how would you go about it? Who would you want to talk to first?

5. If you were President of the United States, what national energy source would you recommend? Why?

Extension Ideas

1. Based on the following information, determine which of 3 locations (Paris, Dallas, or Johannesburg) is best suited for solar power. Then locate each city on a globe. Explain why certain locations are better suited for utilizing solar energy.
   - Day length (in hours: minutes) on December 15 by latitude: 20° = 10:56, 30° = 10:14, 40° = 9:20, 50° = 8:05
   - Day length on June 15 by latitude: 20° = 13:20, 30° = 14:04, 40° = 15:00, 50° = 16:21

2. Create 3 or more identical solar cookers. (For instructions on building a solar cooker, visit: www.solarcooking.org/plans.) Mount each solar cooker at different angles to the horizon. Using the formula from the lesson worksheet, predict the optimum angle. (Note that the formula given is intended to calculate optimum angles for installation in winter in North America.) Use thermometers to measure the temperature at the center of each solar panel after 1 hour at midday on a sunny day. In reality, which angle works best (i.e., gets the hottest)?
3. Create PowerPoint presentations on the pros and cons of various sources of energy, including geothermal, solar, wind, coal, oil, and uranium.

**Additional Resources**

- [www.eia.doe.gov/kids/energyfacts/sources/renewable/solar.html](http://www.eia.doe.gov/kids/energyfacts/sources/renewable/solar.html)—The Energy Kid’s Page from the Energy Information Administration website provides basic information about solar energy, including its advantages and disadvantages. A graphic explains how photovoltaic cells work.

- [www.energyquest.ca.gov/story/chapter15.html](http://www.energyquest.ca.gov/story/chapter15.html)—Chapter 15 of *Energy Story*, an online resource from the California Energy Commission, includes background information about solar energy and photographs of solar mirrors and solar cells.

**Action Project**

Have students evaluate energy use in their homes and at school and devise ways to conserve energy at both locations. Energy information and conservation ideas can be obtained from the Energy Information Administration ([www.eia.doe.gov](http://www.eia.doe.gov)) and the Alliance to Save Energy ([www.ase.org](http://www.ase.org)).
Quick Quiz

1. Coal is a rock that can be used to provide energy for electricity. What is coal made from?
   a) extinct mammals  
   b) ancient plants  
   c) dinosaurs

2. What percent of energy used in the United States comes from renewable energy sources (solar, hydro, geothermal, biomass, and wind)?
   a) 7%  
   b) 17%  
   c) 27%

3. Which of the following provides the most energy to the United States?
   a) solar energy  
   b) firewood  
   c) petroleum oil  
   d) coal

4. Which of the following uses the most electricity in a given period of time?
   a) television  
   b) refrigerator  
   c) vacuum  
   d) laptop computer  
   e) clock radio  
   f) hair dryer
Investigations

1. What kinds of places do you think would be best for using solar energy (energy from the sun)?

2. Solar panels consist of photovoltaic cells, which are devices that turn the sun’s energy into electricity. In order to get the most energy from the sun that you possibly can, you need to position solar panels at a particular angle. The goal is for the sun to hit the panel perpendicularly.

To calculate the optimal angle at which to tilt your solar panel from the horizon, multiply your latitude by 0.9 and add 29 degrees. Find the optimum angle of tilt for the following latitudes, using the formula just given.

<table>
<thead>
<tr>
<th>Latitude</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>5°</td>
<td>33.5°</td>
</tr>
<tr>
<td>20°</td>
<td>47°</td>
</tr>
<tr>
<td>35°</td>
<td>60.5°</td>
</tr>
<tr>
<td>50°</td>
<td>74°</td>
</tr>
<tr>
<td>65°</td>
<td>87.5°</td>
</tr>
</tbody>
</table>

3. The following solar panel is to be installed at 35° latitude using a pole mount. The configuration looks like the following drawing. Angle B is the angle that you calculated for 35° latitude. Find the measure of Angle A.

4. Is m\(\angle B\) acute, right, or obtuse?

5. What angle measurement is the supplement to \(\angle B\)?

ANSWERS

1. Answers may include places that get a lot of sun, latitudes where days are long, and locations closer to the equator.

2. See table.

3. \[m\angle A + 60.5 + 90 = 180\]
   \[m\angle A = 29.5°\]

4. acute

5. \[60.5 + x = 180\]
   \[x = 119.5°\]
6. Is $\triangle ABC$ acute, right, or obtuse?

7. If this solar panel was to be installed at 65° latitude, would that make $m\angle A$ larger or smaller than it was in problem #3?

8. Why do you think solar panels in the Northern Hemisphere are oriented to face south?

9. Use a trigonometric ratio to find the height of the pole from $\angle B$. Round your answer, in degrees, to the nearest tenths place.

10. Use the Pythagorean theorem to find the hypotenuse of $\triangle ABC$. Round your answer, in feet, to the nearest tenth.

**Bonus**

How do you think the angle for tilting a solar panel from the horizon changes in summer months?

---

**ANSWERS**

6. right

7. Smaller because $m\angle B$ would be larger

8. The sun’s rays are most direct at the equator, which is south of North America

9. $\tan \angle 60.5 = \frac{x}{30}$
   
   $x \approx 53.0$ ft

10. $(30)^2 + (53)^2 = c^2$
    
    $c \approx 60.9$ ft

**Bonus**

It is smaller in summer months since the sun’s rays are more directly overhead in summer. The ultimate objective in angling a solar panel is for it to be perpendicular to incoming solar radiation.
Latitude refers to the angular distance from the equator to a particular location in the world. Lines of latitude run parallel to the equator.

1. Below are latitudes (measured in degrees) of 12 cities. The solar energy used to power solar panels (which produce electricity) is best found at midday when the sun is the strongest. Calculate the angle for tilting a solar panel from the horizon that allows the most solar energy to be captured by a solar panel. To calculate this angle for any latitude, multiply the latitude by 0.9 and add 29 degrees.¹

<table>
<thead>
<tr>
<th>Latitude (Cities)</th>
<th>Angle of Solar Panel Tilt</th>
</tr>
</thead>
<tbody>
<tr>
<td>25° (Key West and Taipei)</td>
<td>51.5°</td>
</tr>
<tr>
<td>30° (Houston and Cairo)</td>
<td>56°</td>
</tr>
<tr>
<td>35° (Albuquerque and Tokyo)</td>
<td>60.5°</td>
</tr>
<tr>
<td>40° (Denver and Madrid)</td>
<td>65°</td>
</tr>
<tr>
<td>45° (Minneapolis and Milan)</td>
<td>69.5°</td>
</tr>
<tr>
<td>50° (Winnipeg and Prague)</td>
<td>74°</td>
</tr>
</tbody>
</table>

2. Why do you think the tilt angle is smaller near the equator (at Key West and Taipei)?

3. What angle measurement is complementary to the tilt angle for Winnipeg, Canada?

4. Calculate the angle from the horizon at which each of the following solar panels is mounted:

5. Which angle from the previous problem is near the same latitude as Winnipeg?

**ANSWERS**

1. See table.
2. The sun is more directly overhead, so less tilt is required.
3. 90° − 74° = 16°
4. See drawing.
5. The last one, with a tilt angle of approximately 74°.
Lesson 11

Area & Transformations

Wildlife Habitats

Snow leopards are an endangered species of wildcat that are native to the mountains of central Asia. Students compare the sizes of different habitats snow leopards live in around the world and practice transforming a trapezoid on a coordinate plane. They reflect on what makes this species endangered, characteristics of a proper habitat, and different ways of helping endangered species to survive.
Critical Thinking Questions

- What are ways to conserve and protect Earth’s endangered species?
- How is environmental protection linked to the protection of our livelihoods?

Objectives

- Use formulas to find the areas of polygons
- Transform figures on a coordinate plane
- Analyze ways in which habitat size and location might affect species
- Discuss ways to ensure species survival

Key Concepts

- Geometric figures
- Transformations
- Endangered species
- Habitats
- Biodiversity

NCTM Standards and Expectations Addressed

Geometry: Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life

Measurement: Understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume

Problem Solving: Solve problems that arise in mathematics and in other contexts

Communication: Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Connections: Recognize and apply mathematics in contexts outside of mathematics

Representation: Use representations to model and interpret physical, social, and mathematical phenomena

Materials/Preparation

Post-it notes, 1 per student
(Optional) Picture of snow leopard
Graph paper, 1 page per group of 2-3 students
Student handout: Home Sweet Habitat, 1 per group
Teacher master: Home Sweet Habitat

Background and Practice

Background reading: Helping the Planet and Ourselves
Practice worksheet: Practice with Transformations
Teacher Instructions

1. Pass out 1 Post-it note to each student and ask them to write on their note 1 unique thing that they are able to do. This talent should be something that they are proud of and that they recognize many people are not able to do.

2. Collect the notes and read aloud your students’ skills (without disclosing their names). Place the notes on a wall as you do so.

3. Remove 5 of the notes, reading aloud their contents as you pull them off the wall. Ask students what would happen if no one had these specific skills. What would the world be like if no one possessed these talents?

4. Ask students what happens when certain species of plants and animals disappear from an ecosystem. Often many species are affected when 1 species goes extinct.

5. Ask students what they think the word endangered means. An endangered species is one that is at risk of becoming extinct.

6. Show a picture of the snow leopard (www.snowleopard.org) and explain to students that snow leopards are an endangered species. (Optional: Show the 2-minute snow leopard clip from Discovery Channel: http://dsc.discovery.com/videos/planet-earth-mountains-snow-leopard-hunt.html.)

7. Divide the class into groups of 2-3 students. Give each group a copy of the worksheet, Home Sweet Habitat. Allow them the remainder of the class period to complete the worksheet.

8. If time permits, conclude the class with the discussion questions below.

Discussion Questions

1. What habitat characteristics other than size should be considered when protecting an endangered species?

2. What are some reasons plants and animals may become endangered? Can you name any endangered species?

3. Why do you think it is sometimes difficult for humans and snow leopards (or any predator, such as wolves in North America) to coexist?

4. Can you think of any species that could exist in isolation (that is, a species that doesn’t depend on any other species)?

5. Do you think all species are valuable? Why or why not?
Extension Ideas

1. If you only have enough fencing materials to create a fenced zoo habitat with a perimeter of 200 meters, which of the following habitat designs would provide the greatest area: a) a square with sides of 50m, b) a rectangle with sides 60m and 40m, c) an isosceles triangle with two 75m sides, d) a trapezoid with bases of 60m and 40m and two 50m sides?

2. Research information on the snow leopard, including the size of snow leopards’ natural range. (See www.snowleopard.org and www.snowleopardconservancy.org.) Create a model wildlife refuge for snow leopards built to scale. Based on the range of snow leopards, what is the minimum required perimeter of a refuge? What is the area of the refuge you created? What will you include in the refuge to ensure that the snow leopard is safe?

3. Research the Golden Gate National Recreation Area, an urban wildlife refuge in California (www.nps.gov/goga/index.htm or http://wildequity.org), or a wildlife refuge that is close to where the students live. How many miles does the area include? What endangered species does it include? How is the refuge involving local citizens in saving endangered species and maintaining biodiversity?

Additional Resources

- www.worldwildlife.org—The World Wildlife Fund is one of the largest conservation organizations in the world and has a comprehensive website. Within the website, students can learn about different endangered species, sustainable activities to support the globe, and renewable/nonrenewable resources.

- www.fws.gov/endangered—The U.S. Fish and Wildlife Service has information on the Endangered Species Act, as well as facts about each species classified as threatened or endangered in the United States.

Action Project

Students can visit the Snow Leopard Trust website (www.snowleopard.org) to read about different actions they can take to help support the endangered snow leopard. Just a few possibilities include adopting a snow leopard, sending ecards to spread the word about snow leopard conservation, and purchasing handicrafts from people who have pledged not to harm snow leopards. Have students choose ways to support conservation of snow leopards.
**Investigations**

1. Calculate the area of each of the following 3 habitats of the snow leopard. (Note that the habitats are not drawn to scale.)
   a) The Bronx Zoo, where snow leopards have been living for the past century

   ![Bronx Zoo](image)

   b) Naltar Wildlife Sanctuary in Pakistan, part of the Naltar Valley where snow leopards reside

   ![Naltar Wildlife Sanctuary](image)

   c) Ajar Canyon, a wildlife reserve in Afghanistan where a few snow leopards reside

   ![Ajar Canyon](image)

2. What observations can you make about the relative sizes of the 3 habitats?

3. Suppose the Bronx Zoo decides to increase the snow leopard area to make it 3 times larger. What is the area of the new habitat?

4. Suppose that instead of making the zoo habitat 3 times larger, the Bronx Zoo decides to make each side of the habitat 3 times longer. Would this produce a larger or smaller habitat than your answer to #3?

5. In what ways might increasing the size of a habitat affect an animal?

6. On graph paper, graph a trapezoid with vertices E (1,1), F (16,1), G (21,15), and H (-3,15). This trapezoid represents the Naltar Wildlife Sanctuary in Pakistan.

**ANSWERS**

1. a. \( A = 0.04 \times 0.1 \)
   \[ = 0.004 \text{ km}^2 \]
   b. \( A = \frac{1}{2}(14)(24 + 15) \)
   \[ = 273 \text{ km}^2 \]
   c. \( A = \frac{1}{2}(500)(160) \)
   \[ = 40,000 \text{ km}^2 \]

2. The snow leopard at the zoo has an extremely small space compared to areas in Pakistan and Afghanistan.

3. \( 0.004 \times 3 = 0.012 \text{ km}^2 \)

4. The area of the habitat will be larger if the side lengths are all 3 times longer: \( 0.04(3) + 0.1(3) = 0.036 \text{ km}^2 \)

5. Answers will vary: If the animal is used to roaming large areas of land, a larger space may allow them to express more typical movements and behaviors. The larger space could mean less conflict between animals that share a habitat. A larger habitat might include more resources such as shade, shelter, or food.
7. Developers want to build several hotels in part of the wildlife sanctuary to encourage tourism. However, a law requires that the same number of acres remain a protected wildlife refuge. To make room for the hotels but retain the same size refuge habitat, the borders of the protected region will need to move 4 km to the west and 8 km to the south. Use the formulas \( x = 4 \) and \( y = 8 \) to translate the vertices of your trapezoid. Draw this new trapezoid on your graph, using a different color than your original trapezoid.

8. Would it make sense to move a protected refuge from an area where snow leopards currently reside to a new area? Explain why you think it is a good idea or a bad idea.

9. Funding to maintain the protected sanctuary in Pakistan has been cut in half. The Parks Director is in charge of reducing the protected area by half. She draws the same trapezoid on graph paper that you did in problem #6, with vertices E (1,1), F (16,1), G (21,15), and H (-3,15). She decides to multiply each coordinate by \( \frac{1}{2} \) in order to produce a trapezoid half the size of her original figure. Draw this new trapezoid on your graph, using a different color than your original trapezoid.

10. Was the Parks Director successful in producing a new trapezoid whose area is half the area of the original trapezoid? Explain your answer.

**Bonus**

A snow leopard sees an ibex (a wild mountain goat) at the top of a mountain. The ibex is one of the snow leopard’s favorite foods. The height of the mountain is 8,000 feet and the base of the mountain is 12,000 feet across. The base is bisected by the mountain’s highest point. If the snow leopard is at the foot of the mountain, what is the distance it will need to run in order to reach the ibex?

**ANSWERS**

6. [Diagram of trapezoids]

7. See graph (dashed trapezoid). New coordinates will be E (\(-3, -7\)), F (12, -7), G (17, 7) and H (\(-7, 7\)).

8. Possible answers:

   - It is a good idea because the hotels will bring more money and visitors to the region, which might indirectly help the snow leopards; the snow leopards will still have the same amount of area to roam; it will be a protected area where the snow leopards are safe.

   - It is a bad idea because there may not be the same quantity or quality of resources in the new area; the new location may have different environmental conditions (climate, altitude, etc.).

9. See graph (smallest trapezoid). If all vertices are multiplied by \( \frac{1}{2} \), the new vertices are E (0.5,0.5), F (8,0.5), G (10.5,7.5), and H (\(-1.5,7.5\)).

10. No, the new area is 68.25 km², which is actually \( \frac{1}{4} \) the original area.

**Bonus**

A bisected base infers that it will be divided into 2 equal halves. The distance can be found using the Pythagorean theorem: \( a^2 + b^2 = c^2 \).

\[
(8000)^2 + (6000)^2 = c^2
\]

\[
c = 10,000 \text{ ft}
\]
A transformation involves the movement of a geometric figure. There are several different kinds of transformations.

One type of transformation is a translation, which causes a geometric figure to "slide" from one location to another. All vertices move the same distance in the same direction.

1. Describe how the $x$ and $y$ values in the example above change from the initial location to the new location.

2. Suppose a triangular piece of land in the Amazon Rainforest in Brazil is protected from logging. In the following coordinate plane, graph a triangle with vertices A (3, 5), B (5, 1), and C (1, 1) to represent this piece of land.

3. After the trees in the area have grown back, the triangular piece of land will be logged and a different triangular area of land to the southwest will be protected instead. Using the formulas $[x - 6]$ and $[y - 3]$, translate the triangle to a new area on the coordinate plane so that the triangle moves 6 units left and 3 units down. Draw this new triangular piece of protected land on the same graph, using a different color than your original square.

4. In the example above, both coordinates for each vertex of the larger square were multiplied by $\frac{1}{2}$ to produce the smaller square. Draw a larger figure on the same graph by multiplying each of the larger square's coordinates by 2.

**ANSWERS**

1. $x$ moves right 3 units, and $y$ moves down 7 units
2. See graph.
3. See graph (dashed triangle).
4. See graph (dashed square).
   Coordinates: (4,4), (4, -4), (-4,-4), (-4,4)
Students analyze the characteristics of sustainably designed items. In a real-world application exercise, students calculate the surface area and volume of a prism and a cylinder in order to connect geometric properties to sustainable design principles. A closing discussion raises questions about the applicability of sustainable design to different situations, as well as its social and economic impacts.
Critical Thinking Questions

• How is the design of an object tied to creating a well-balanced environment, society, and economy?
• What are objects you could envision being redesigned?

Objectives

• Calculate surface area and volume of cylinders and prisms
• Use geometric measurements to improve the sustainability of product design
• Investigate resource use as one consideration of sustainable design

Key Concepts

• Surface area of prism and cylinder
• Volume of prism and cylinder
• Sustainable design
• Fuel efficiency

NCTM Standards and Expectations Addressed

Geometry: Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties

Geometry: Use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume

Geometry: Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life

Measurement: Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision

Problem Solving: Solve problems that arise in mathematics and in other contexts

Reasoning and Proof: Make and investigate mathematical conjectures

Communication: Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Connections: Recognize and apply mathematics in contexts outside of mathematics

Representation: Use representations to model and interpret physical, social, and mathematical phenomena

Materials/Preparation

Overhead: Milk Jug Designs
6 - 1 gallon milk jugs
6 crates or boxes of equal size that could fit at least 4 milk jugs (such as a paper box)
Handout: Taking Shape, 1 per student
Teacher master: Taking Shape

Background and Practice

Student reading: Designer Products
Practice worksheet: Practice with Surface Area & Volume
Teacher Instructions

1. Ask students what kinds of things they use every day that someone has designed. *There are infinite answers here—backpacks, cups, pens, cars, etc.*

2. Challenge students to think of how the design of a particular item or building suits its use. *For example, a sturdy shopping bag with a reinforced bottom is suitable for heavy objects. A building with south-facing windows is ideal for receiving natural light.* Can they think of examples of poorly designed objects? *For example, a lotion container that makes it difficult to get the lotion out of the container, or a small bathroom stall where the door opens inward.*

3. Ask students to name a 3-dimensional geometric figure that resembles a plastic milk jug. *Some might say it resembles a cylinder or a prism.*

4. Split students into groups of 3-4 students.

5. Hand out a milk jug and a crate or box to each group.

6. Read the following scenario to them: “Your group is part of a milk distribution company. You are trying to figure out how to make the transport of milk from dairy farms to stores across the nation as cost-efficient and environmentally-responsible as possible. Your first task is to determine how many 1-gallon milk jugs can fit into the crate you were given.”

7. Give groups 1 minute to estimate how many of their milk jugs could fit into their crate/box.

8. Ask students if there would be any unused space in the crate/box if it was filled with milk jugs. What factors contribute to this wasted space? *Jugs cannot be stacked on top of each other; the handle seems to be wasted space; the neck is narrower than the body of the jug.*

9. Ask students how the use of milk jugs and crates/boxes like these is related to the cost-effectiveness of transporting milk from a dairy farm to a store. *Because of wasted space, more trips might have to be made; small crates mean that more crates need to be used.*

10. Give groups 1 final minute to brainstorm how the milk jug could be redesigned so that more could fit onto a truck.

11. Ask a representative from each group to share his or her group’s ideas.

12. Explain to the class that they just did an exercise in **sustainable design**. *Sustainable design refers to designing products and buildings with sustainability in mind—that is, maximizing economic benefits while minimizing negative impacts on people and environmental resources.* **Sustainable design can be used to design almost anything, including milk jugs!**
13. Pass out a *Taking Shape* handout to each student. You may want to allow students to work together in pairs. If they are unsure of the formulas for calculating volume and surface area, share the following formulas with them:

- cylinder volume:
  \[ V = \pi r^2h \]
- prism volume:
  \[ V = Bh, \text{ where } B \text{ is the area of the base} \]
- cylinder surface area:
  \[ SA = 2\pi r^2 + 2\pi rh \]
- prism surface area:
  \[ SA = 2(\text{area of base}) + 2(\text{area of front side}) + 2(\text{area of right side}) \]

14. Note that for several of the questions on the handout (1, 5, 8, 9, and 10), there will likely be a variety of answers. The objective with these questions is for students to think critically about the link between geometry and sustainable design.

15. After students complete the worksheet, lead a class discussion using the following questions.

**Discussion Questions**

1. In 2008 Sam's Club began to use a new milk jug design, shaped somewhat like a rectangular prism, to cut down on shipping costs. More of the new milk jugs can fit in a single truck, resulting in fewer trips. Not everyone was happy about the switch—customers complained that it is difficult to pour the milk without the handle and narrow neck that the old jugs had. How could Sam's Club help people adjust to using the new containers?

2. Why do you think sustainable design has to consider the economic and social effects of new designs, rather than just the environmental impacts?

3. Would you buy sustainably designed items even if they looked very different than what you are used to? Why or why not?

4. Would you pay more for products that were designed with sustainability in mind?
Extension Ideas

1. Sam’s Club sells gallons of milk in plastic containers similar to the cylinder-based design. They claim that it has dropped the cost of a gallon of milk from $2.58 to $2.18. Calculate the percent savings to the customer for a gallon of milk sold in the new containers.¹

2. If the new containers sold by Sam’s Club contain 4.5 gallons of milk for every 1 ft³ of truck space, which is 50% more than the old containers,² how many gallons of milk per cubic foot did the old jugs hold?

3. How many of the new milk jugs (13 cm x 13 cm x 25 cm) will fit in a truck whose trailer is 28 feet long, 10 feet high, and 10 feet wide?

   Hint: The measurements given for the jugs are in different units than the truck measurements. In order to work with the same units, you will first need to convert ft to cm, or cm to ft.

4. Can you design an even more efficient container than the prism and cylinder designs? Using clay, try designing containers based on other polyhedron shapes, such as pyramids or cones. The objective is to reduce wasted space on trucks that ship the milk from dairies to grocery stores.

Additional Resources


- www.usgbc.org — The U.S. Green Building Council provides information about green building certification (the Leadership in Energy and Environmental Design rating system, or LEED). Their website includes an Educator Resource Center.

Action Project

Have students research products available in their community that they consider sustainable or sustainably designed. Put together a guide alerting citizens to where they can find these products. Restaurants and grocers that sell sustainably produced food (such as Fair Trade coffee or organically grown produce) could also be included in the guide. Distribute the guide at local shops and restaurants that sell these products.
Milk Jug Designs

Traditional Milk Jug

Redesigned Milk Jug
**Investigations**

1. We are going to examine 2 milk container designs to see which one is more sustainable. But first, write down at least 3 ways that the concept of sustainable design could be used to redesign plastic milk jugs. **Sustainable design** involves making products in a way that minimizes negative impacts of production on people and environmental resources while still balancing economic costs.

2. Calculate volume for the following 2 containers:

   ![Container A](image1)
   ![Container B](image2)

3. Which container holds a greater volume of milk?

4. What feature(s) of this container allows it to hold more milk?

5. How is a container’s volume related to sustainable design?

6. Now calculate and compare the surface area of the 2 containers. Round answers to 2 decimal places (the hundredths place).

7. Which container has a greater surface area?

8. How is surface area related to sustainable design?

9. Of the 2 containers, which one would you recommend stores carry? Take into consideration volume, surface area, production and transport costs, the environment, and usability.

10. What other design features would you add to the preferred container to make it even easier to use? Explain how these features would affect the sustainability of the container’s design, either positively or negatively.
ANSWERS

1. Just a few possibilities include:
   • reducing volume so that containers take up less space in store refrigerators (saving energy) and less space in delivery trucks (reducing fuel use)
   • reducing surface area so that less packaging is required (saving resources)
   • using recycled plastic
   • making sure the container can be easily recycled or reused

2. Container A
   \[ V = \pi r^2 h \]
   \[ V = \pi (6.5)^2 (25) = 3318.3 \text{ cm}^3 \]

3. Container B
   \[ V = Bh \]
   \[ V = (13)(13)(25) = 4225 \text{ cm}^3 \]

4. Container B has a larger base area, resulting in greater volume.

5. A container with more volume holds more milk, reducing the number of containers (and often the amount of packaging) used.

6. Container A
   \[ SA = 2\pi r^2 + 2\pi rh \]
   \[ = 2\pi (6.5)^2 + 2\pi (6.5)(25) \]
   \[ = 265.46 + 1021.02 \]
   \[ = 1286.48 \text{ cm}^2 \]

   Container B
   \[ SA = 2(13 \cdot 13) + 2(25 \cdot 13) + 2(25 \cdot 13) \]
   \[ = 338 + 650 + 650 \]
   \[ = 1638 \text{ cm}^2 \]

7. Container B

8. A large surface area requires more packaging materials (plastic, in this case). Items with a small surface area could be considered more sustainable.

9. Possible answer: Stores should carry Container B. Even though its surface area is larger, it has a greater volume, and the containers can be stacked on top of each other. This means more containers can fit into a truck. Less fuel would be used to transport the containers because delivery trucks wouldn’t have to make as many trips.

10. Features such as handles and a narrow neck for pouring would make the container easier to use, but would likely diminish the environmental and economic sustainability of the container. Another idea is to fashion the container from material that insulates the milk better so that the refrigerator doesn’t need to be kept as cold. This would improve the sustainability of the container. The sky is the limit with sustainable design ideas!
**Surface area** is the sum of the areas of the faces or surfaces of a 3-dimensional object.

The area of any side of the above cube is 4 in$^2$. 
(2 in $\times$ 2 in = 4 in$^2$)

In the case of a cube, all 6 sides have the same area.

The surface area can be found by adding together the areas of all sides:

\[4 \text{ in}^2 + 4 \text{ in}^2 + 4 \text{ in}^2 + 4 \text{ in}^2 + 4 \text{ in}^2 + 4 \text{ in}^2 = 24 \text{ in}^2\]

1. Juice cartons can be recycled into notebook covers. Calculate the surface area of the following juice carton in square centimeters (cm$^2$).

2. A company wants to make small notebooks out of juice cartons. The notebooks have front and back covers that are each 15 cm long and 10 cm wide. How many complete notebooks can be made from a single juice carton?

**Volume** is the number of cubic units (such as cm$^3$) that are needed to fill a 3-dimensional figure.

For a cube or prism, the volume (V) is the base area ($B$) multiplied by the height ($h$).

In this example, $B = 2 \text{ in} \times 2 \text{ in} = 4 \text{ in}^2$.

\[V = Bh = 4 \text{ in}^2 \times 2 \text{ in} = 8 \text{ in}^3\]

That means that 8 cubic inches can fit inside the cube.

3. A different company wants to devise a way to recycle the juice cartons, sanitize them, and refill them. What is the maximum volume (in cubic centimeters, or cm$^3$) of liquid that each carton can hold?

**ANSWERS**

1. \[\text{SA} = 2(90.25) + 4(180.5) = 902.5 \text{ cm}^2\]

2. \[
\begin{align*}
A &= 15 \times 10 = 150 \text{ cm}^2 \\
902.5 \text{ cm}^2 &\div 150 \text{ cm}^2 \approx 6.02 \\
&= 3 \text{ notebooks}
\end{align*}
\]

3. \[V = 9.5 \text{ cm} \times 9.5 \text{ cm} \times 19 \text{ cm} = 1714.75 \text{ cm}^3\]
This 2-day lesson begins with a lively introductory activity in which students examine how a system works. They experience limits to success, redesign a system to improve efficiency, and begin to identify assumptions that drive human behavior. Then students analyze current global trends in primary school education and use systems thinking to generate solutions that address root causes of barriers to education.
Critical Thinking Questions
• How can understanding the nature of systems help us find solutions to large, complex problems?
• How can we redesign a system to achieve a desired outcome?
• What part does education play in creating a stable future for the entire world?

Objectives
• Graph linear functions
• Calculate slope
• Discover worldwide trends in primary school completion
• Use systems thinking to look at problems

Key Concepts
• Linear functions
• Slope
• Systems thinking
• Primary education

NCTM Standards and Expectations Addressed
Algebra: Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules
Algebra: Identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations
Algebra: Explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope

Algebra: Model and solve contextualized problems using various representations, such as graphs, tables, and equations
Algebra: Use graphs to analyze the nature of changes in quantities in linear relationships
Problem Solving: Solve problems that arise in mathematics and in other contexts
Reasoning and Proof: Make and investigate mathematical conjectures
Communication: Communicate mathematical thinking coherently and clearly to peers, teachers, and others
Connections: Recognize and apply mathematics in contexts outside of mathematics
Representation: Use representations to model and interpret physical, social, and mathematical phenomena

Materials/Preparation—Day 1
Stuffed bear or other soft object suitable for tossing
Watch with a second hand or a stop watch
Open space for introductory activity
Student handout: Bears in the Air, 1 for each student
Teacher master: Bears in the Air
Graph paper, 1 page per student
**Materials/Preparation—Day 2**

**Student handout:** *Making the Grade*, 1 per pair of students

**Teacher master:** *Making the Grade*

**Graph paper,** 1 page per pair

**Blue and red pens,** 1 each per pair

**Background and Practice**

**Background reading:** *Education for All?*

**Practice worksheet:** *Practice with Linear Functions*

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**Teacher Instructions—Day 1**

1. In a large open space, arrange students so that they stand shoulder-to-shoulder in a circle. Stand in the circle with them and show them the stuffed bear or other soft object for tossing.

2. Tell students they are going to play a game in which they toss the bear around the circle. There are only 2 rules: (1) Everyone must touch the bear, and (2) Students must touch it in the same sequence each time they play.

3. Have everyone hold their hands out, ready to catch the bear.

4. Gently toss the bear to someone across the circle.

5. Ask that student to toss the bear to someone else and then drop his or her hands. Continue until everyone has touched the bear once. The last person tosses the bear back to you.

6. Practice once so that students are comfortable repeating the sequence.

7. Now tell the group that you will time the activity to see how fast they can do it.

8. Run the activity and time it. Write on the board the amount of time it took students to complete the round. After this first round, tell students that you are sure they could do it faster.

9. Run and time the activity for 4 additional rounds, recording the time students take to complete each round. Once students reach a certain level of success, they will not get any faster without a system redesign. In fact, they may get sloppy and drop the bear or toss it too far in their attempt to go faster. This part of the activity models the concept of **limits to success.** Following a certain system may get you specific results, but there are limits to success based on the system you create.

10. Pass out the handout, *Bears in the Air.* Have students fill in the times for trials 1-5 on their worksheets.
11. Give students about 15 minutes to complete numbers 1-4 on the worksheet. To find slope for question #3, use the formula: $\text{slope} = \frac{\text{rise}}{\text{run}}$, or $\text{slope} = \frac{\text{(difference in } y\text{-coordinates)}}{\text{(difference in } x\text{-coordinates})}$. To estimate the time the 8th trial will take for question #4, assign a value of 0 to the $y$-intercept ($b$) in the formula, $y = mx + b$.

12. Students may ask, or you may suggest, whether the arrangement (standing in a circle) may be changed. The answer is yes. Repeat the 2 rules of the game, and invite students to suggest different designs. Now test at least 3 redesigns, 1 at a time. (Student inventions might include standing next to each other and passing the bear along the line, or creating a cascade of hands in the correct sequence, then letting the bear slide down the cascade.) This part of the activity models the concept of system redesign. When you redesign a system, the system could potentially get better.

13. Continue until students have improved their time and arrived at a favorite solution. This part of the activity models the concept of testing alternative designs.

14. Have students record their times for all trials on their Bears in the Air worksheet and complete the worksheet.

15. Review the answers together as a class, and conclude with a discussion using 1 or more of the following questions.

**Discussion Questions—Day 1**

1. What happened the first few times through? Did you succeed in doing the activity faster? Why?

2. Did anyone think about other ways of doing it, but did not speak up? What kept that person from offering a solution?

3. Did anyone offer a solution that was ignored? Why was their solution ignored?

4. Can you think of some real-world examples (past or present) in which people experience the limits to success by doing something harder or faster? What are the assumptions people hold about how a particular system functions? How might that system be redesigned to achieve a common goal?
Teacher Instructions—Day 2

If teacher decides to omit Day 1 activities, instructions 1–3 are another way of introducing the concept of systems.

1. Ask students what people/positions make up a basketball team. (e.g., point guard, center, forwards, guards, a coach, etc). Ask: “If a point guard is taken out of the team, what happens? If the coach disappears, what happens?”

2. Explain that the basketball team is a system of people that need various things to make it work. If any part of the team is removed, the entire team is affected. The system does not function in the same way when 1 part is removed or added.

3. Ask students if they can name any other systems. Just a few examples include a government, a car, a computer, the human body, and the solar system.

4. Explain that primary education is another example of a system. Primary education is often called “elementary education.”

5. Ask them to brainstorm who/what is involved in this system. (e.g., students, teachers, schools, districts, government)

6. Explain to students that many children around the world are not able to attend school. Ask students to guess the number of children around the world that do not attend school. (approximately 80 million children) As a follow-up question, ask: “What might be reasons some children aren’t attending school?” Some may drop out, some may live in towns or villages with no school, some are too sick, some may have to work to help support their families.

7. Tell students that they will work with a partner, then assign each student a partner. Pass out the handout, Making the Grade, to each student pair. Each pair of students will also need 1 piece of graph paper, 1 red pen, and 1 blue pen.

8. Assign each group a specific region (Africa, South Asia, Latin America/Caribbean, or Europe/Central Asia) to investigate. They will explore education statistics for their given region.

9. You may want to have at least 1 pair from each region share their findings for their particular region with the class so that students can see global trends.

10. After students have completed the handout, explore the issue of inequitable access to education using the following discussion questions.
Discussion Questions—Day 2

1. What are some consequences of *not* completing a primary school education? How would your life be different if you were unable to attend school?

2. What part does education play in shaping the future? What might be some consequences of *not* encouraging education worldwide?

3. Why do you think the Millennium Development Goals for the 4 different regions studied (Africa, South Asia, Europe, and Latin America/Caribbean) are different than the actual numbers of students completing primary school?

4. Can you think of a system redesign that would increase the number of students attending school in a region?

Additional Resources

- [www.campaignforeducation.org](http://www.campaignforeducation.org)—Students can learn more about the Global Campaign for Education and participate in the World’s Largest Lesson by signing up online.


Extension Ideas

1. Create a resolution which states that 100% of students enrolled in the region you studied will complete primary school education. Formulate a linear equation in order to determine 5-year targets for increased attendance. Create a line graph that extends to the year in which you resolve that all students from your region will complete school.

2. Many statistics are published annually by the National Center for Education Statistics ([www.nces.ed.gov](http://www.nces.ed.gov)). Study education statistics and trends for the United States. How has the dropout rate changed during the last 50 years? How has the percentage of students attending college changed? Are any of the trends linear? If so, create a linear equation to represent a linear increase or decrease. Graph the observed trends.

Action Project

Have students identify a local system in their school community (e.g., the school, classroom, the school district, watershed, or playground) that is in need of reform. They can then create a proposal that would help to transform this system to make a positive difference. Have students present their proposals to decision-makers in the school community.
Investigations

1. In the chart below, fill in the time it took you to complete each trial:

<table>
<thead>
<tr>
<th>Trial (x)</th>
<th>Time, in Seconds (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

2. Plot points from Trial 1 and Trial 2 on a coordinate plane.

3. Calculate the slope of the line formed by these 2 points.

4. Based on the slope you just calculated, what would the time of the 8th trial be, assuming your progress proceeds linearly?

5. Plot points for the remaining trial results on the same graph where you plotted Trials 1 and 2.

6. How does the actual time recorded for Trial 8 compare to the time you estimated based on slope (problem #4)? If they are different, why do you think they are different?

7. Do you think there is a limit to how fast you could complete the tossing activity? Why, or why not?

ANSWERS

1. Answers will vary.

2. Answers will vary.

3. Slope = \( \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \)

4. Students can use the slope to plot what the 8th trial would be, using the slope-intercept form: \( y = mx + b \).

5. Answers will vary.

6. It is likely that the actual results will not match the original estimate. The system was redesigned to attempt to shorten the time required. Class results will not indicate a linear function.

7. Answers will vary.
**Making the Grade**

**Investigations**

You will be investigating trends in primary education (grades K–8) for a certain region in the world.

The United Nations has developed **Millennium Development Goals** to reduce extreme poverty. One of the goals is to achieve universal primary education, meaning every child in the world is able to complete primary school.

The chart below shows the target percentages of primary school completion in 4 regions around the world—if the Millennium Development Goal on education is to be achieved by 2015.1

<table>
<thead>
<tr>
<th>Region</th>
<th>2000</th>
<th>2005</th>
<th>2010</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>Africa</td>
<td>54%</td>
<td>70%</td>
<td>78%</td>
<td>92%</td>
</tr>
<tr>
<td>South Asia</td>
<td>71%</td>
<td>83%</td>
<td>90%</td>
<td>94%</td>
</tr>
<tr>
<td>Latin America / Caribbean</td>
<td>76%</td>
<td>82%</td>
<td>88%</td>
<td>92%</td>
</tr>
<tr>
<td>Europe/Central Asia</td>
<td>92%</td>
<td>94%</td>
<td>96%</td>
<td>98%</td>
</tr>
</tbody>
</table>

1. Create a line graph in blue representing your assigned region's projected percentages from 2000 to 2015.

2. Does your graph indicate a linear increase in the region's percentages over a given time? How do you know whether the data you graphed indicate a linear trend or not?

3. Here are the estimated percentages of primary school completion, based on completion rates in the 1990s:

<table>
<thead>
<tr>
<th>Region</th>
<th>2000</th>
<th>2005</th>
<th>2010</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>Africa</td>
<td>54%</td>
<td>56%</td>
<td>58%</td>
<td>60%</td>
</tr>
<tr>
<td>South Asia</td>
<td>71%</td>
<td>74%</td>
<td>77%</td>
<td>80%</td>
</tr>
<tr>
<td>Latin America / Caribbean</td>
<td>75%</td>
<td>74%</td>
<td>73%</td>
<td>72%</td>
</tr>
<tr>
<td>Europe/Central Asia</td>
<td>92%</td>
<td>93%</td>
<td>94%</td>
<td>95%</td>
</tr>
</tbody>
</table>

On the same graph where you charted your specific region's targets for achieving 100% primary school completion by 2015, graph the completion rates for your region from the second table with a red pen.

4. Explain in 1 sentence what the 2 lines on your graph indicate.

5. Using the data from the table in #3, estimate what percentage of students will complete primary school in your region by the year 2015 and write this number in the last column of the table.

6. In order to meet the goal of 100% primary school completion, does the education system in your region need a redesign? Explain your thinking.

7. The percentage of students in the United States who complete primary school is 94%.2 How does that percentage compare to the estimated percentage of students in your region who will complete primary school in 2010?

**Bonus**

Based on the actual trends observed (shown in the second data table), by what year would 100% of students from each region complete primary school?

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1. **Millenium Development Goal: Primary School Completion Targets**

   - **Africa**
   - **South Asia**
   - **Latin America/Caribbean**
   - **Europe/Central Asia**

2. Only Europe/Central Asia follows a linear increase (i.e., a constant increase over time). The other 3 regions do not follow a linear trend because the rate of increase is not constant. The points do not form a straight line from 2000 to 2015. Therefore, if you calculate the slope between any 2 points on the graph, you will find that no 2 slopes are identical.

3. **Actual Progress toward Primary School Completion by Region**

4. The actual numbers are lower than the goals. Targets are not being met on time.

5. See table.

6. Possible answers:
   - **Africa**—Education system needs a redesign in order to reach goal faster.
   - **South Asia**—Education system in South Asia does not necessarily need a redesign, though progress toward the goal is fairly slow.
   - **Latin America/Caribbean**—Education system in Latin America needs a redesign because primary school completion is gradually declining.
   - **Europe/Central Asia**—Education system does not appear to need a redesign.

7. **Africa**—Completion of primary school is 36% higher in the United States.
   - **South Asia**—Completion is 17% higher in the United States.
   - **Latin America/Caribbean**—Completion is 21% higher in the United States.
   - **Europe/Central Asia**—There is no difference.

**Bonus**

- **Africa**—2115
- **South Asia**—2050
- **Latin America/Caribbean**—Since the numbers are decreasing, an estimate cannot be made based on given data.
- **Europe/Central Asia**—2040
A **linear function** follows the form, \( y = mx + b \).

- \( m \) is the **slope** of the line (a number that indicates how steeply a line goes up or down)
- \( b \) is the **y-intercept** (the point where the line crosses the y-axis)

1. A community wants to increase the percentage of students graduating from high school. They decide to hire more school counselors to assist high school students. The community’s progress toward their goal can be described by the following function:

\[ y = 4x + 67, \text{ where } x \text{ is the number of new counselors hired and } y \text{ is the percent of seniors who graduate each year} \]

Complete the following table using the function \( y = 4x + 67 \):

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>71</td>
</tr>
<tr>
<td>2</td>
<td>75</td>
</tr>
<tr>
<td>3</td>
<td>79</td>
</tr>
<tr>
<td>4</td>
<td>83</td>
</tr>
<tr>
<td>5</td>
<td>87</td>
</tr>
</tbody>
</table>

2. How many new counselors would need to be added to ensure that 99% of seniors graduate?

3. Graph the linear function.

4. What is the percent of seniors that will graduate if no new counselors are hired? (Note: this is the y-intercept.)

5. What is the slope of the function? Explain the function’s slope in the context of counselors and graduation rates.

**ANSWERS**

1. See table.
2. \( 99 = 4x + 67 \)
   
   \[ x = 8 \]

3. 

4. \( 4(0) + 67 = 67\% \) percent will graduate

5. The slope is 4. For each counselor hired, there is a 4% increase in the percentage of seniors graduating.
By plotting the geographical coordinates of community resources in a given city, students will consider how the location of resources affects sustainability. Students calculate distances and midpoints using coordinate geometry.
Critical Thinking Questions
• How does access to resources create a stronger community?
• What decisions can communities, neighborhoods, and cities make to ensure their inhabitants have a more sustainable lifestyle?

Objectives
• Plot distances between different locations on a coordinate plane
• Use distance and midpoint formulas to analyze community resource distribution
• Determine how resource distribution affects community sustainability

Key Concepts
• Distance formula
• Midpoint formula
• Graphing on a coordinate plane
• Sustainable community
• Resource distribution

NCTM Standards and Expectations Addressed
Number and Operations: Work flexibly with fractions, decimals, and percents to solve problems
Number and Operations: Understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems

Algebra: Model and solve contextualized problems using various representations, such as graphs, tables, and equations
Algebra: Use graphs to analyze the nature of changes in quantities in linear relationships
Problem Solving: Solve problems that arise in mathematics and in other contexts
Communication: Communicate mathematical thinking coherently and clearly to peers, teachers, and others
Connections: Recognize and apply mathematics in contexts outside of mathematics
Representation: Use representations to model and interpret physical, social, and mathematical phenomena

Materials/Preparation
Student handout: Putting Sustainability on the Map, 1 per student pair
Teacher master: Putting Sustainability on the Map
Graph paper, 1 page per student pair
Red pens, 1 per student pair

Background and Practice
Background reading: Communities that Last
Practice worksheet: Practice with Midpoint & Distance
Teacher Instructions

1. Ask students what they think a resource is. Explain that it can be a person, place, or thing that is a source of support (e.g., a library or a teacher).

2. Ask students to imagine themselves living in their dream neighborhood 20 years from now.

3. Have them talk to a partner for a couple of minutes and agree to 5 resources they would want their neighborhood to offer that would support everyone (from small children to the elderly).

4. Ask each pair to share the resources they chose. Write these on chart paper or on a board so that everyone can see them.

5. Ask students to point out any trends that they notice based on the list of resources. Are there certain resources that everyone agrees are important to have in a neighborhood?

6. Explain that the resources they chose are ones that make a neighborhood sustainable. In other words, these are resources that assist people in leading healthy and productive lives.

7. Ask them how the neighborhoods they live in now compare to their resource list. Why are certain resources not included in their real neighborhoods?

8. Explain to students that they will be investigating resource distribution in a community in order to decide where to locate a proposed park.

9. Divide the class into pairs and provide each pair with the worksheet, Putting Sustainability on the Map. Make sure each student group also has graph paper and a red pen.

10. For question 2, have students use the midpoint formula: \( \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \).

11. After students complete the handout, lead a discussion using the following questions.

Discussion Questions

1. Is your community or neighborhood designed for people with cars or is it easy to live without a car?

2. What do you think makes a neighborhood “resource rich”? Explain that there’s a difference between a rich neighborhood that has lots of money and a resource-rich neighborhood that offers a lot of services to its community.

3. What does it mean for a neighborhood to have social resources? Economic resources? Environmental resources?

4. Why do you think not all groups of people have equal access to community resources such as parks and public libraries?

5. How do you think people in a community can actively work together to become resource rich?
Extension Ideas

1. Using Google Maps (www.maps.google.com), calculate the distance between your home and the nearest locations of the following resources: a public library, a movie theater, a grocery store, and the school you attend. Would you consider where you live a model for sustainable community development? Would you prefer that any of these resources were closer? Are there any resources in your neighborhood that you wish were not there?

2. Based on the state you live in, choose 1 urban neighborhood and 1 rural community to study. Go to www.walkscore.com and analyze which place has more resources that are in walking distance. Compare the resources to see which location seems more sustainable.

Additional Resources

- www.planning.org—The American Planning Association (APA) is a nonprofit public interest and research organization committed to urban, suburban, regional, and rural planning. On their site you will find information for professional planners, educators, and students about community planning. They also publish ResourcesZine, an online newsletter with articles and ideas for involving youth in planning efforts.

- www.epa.gov/compliance/wherelyoulive—The U.S. Environmental Protection Agency provides information about current environmental justice issues for each of its 10 regions.

- www.walkscore.com—Walk Score provides users with a measure of how many amenities are located within walking distance of a given address. Type in any address and find nearby grocery stores, movie theaters, schools, fitness facilities, and more.

Action Project

Choose 1 local community issue of interest, such as health care. Identify and map resources related to this issue in your community (for health care, resources might include pharmacies, hospitals, and medical clinics). One tool for finding these resources is Google Maps (www.maps.google.com). Explore the implications of where these resources are located. How might the entire community be affected by the location of a resource? How is the surrounding area affected? Generate ideas for enhancing existing resources, indicating on your map where these enhancements should be located. Work with a neighborhood association or the city planner’s office to make your vision come to life.
Investigations

1. On a piece of graph paper, use a pencil to plot the following locations of community resources found in Greenpoint, USA, and label them to indicate the resource represented by each point.

<table>
<thead>
<tr>
<th>Resource</th>
<th>(x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public library</td>
<td>(2, 4)</td>
</tr>
<tr>
<td>Landfill</td>
<td>(8, 9)</td>
</tr>
<tr>
<td>Hospital</td>
<td>(4, 6)</td>
</tr>
<tr>
<td>Jail</td>
<td>(7, 1)</td>
</tr>
<tr>
<td>Airport</td>
<td>(4, 8)</td>
</tr>
<tr>
<td>Elementary school</td>
<td>(6, 7)</td>
</tr>
<tr>
<td>High school</td>
<td>(4, 7)</td>
</tr>
</tbody>
</table>

2. Use the midpoint formula to determine the midpoint between the elementary school and the high school.

3. Why might you want to find the midpoint between any 2 community resources?

4. Your community voted to build a large recreational park, and now the city government is trying to decide where it should be located. There are 2 vacant lots where it could be built. Plot the following 2 points on your graph using a red pen and label them: A (1, 8) and B (5, 1).

5. Use the distance formula to calculate the following distances (in miles) from both possible park locations to key community resources. Round your answers to 2 decimal places (the hundredths place) and record them in the following table.

<table>
<thead>
<tr>
<th>Community Resource</th>
<th>Future Park Location A (1, 8)</th>
<th>Future Park Location B (5, 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elementary school</td>
<td>5.10 miles</td>
<td>6.08 miles</td>
</tr>
<tr>
<td>Jail</td>
<td>9.22 miles</td>
<td>2.00 miles</td>
</tr>
<tr>
<td>Hospital</td>
<td>3.61 miles</td>
<td>5.10 miles</td>
</tr>
</tbody>
</table>

6. Based on the distances you calculated, would you choose location A or B for the new park? Explain your answer.

7. List at least 3 resources that would make a community more environmentally, economically, or socially sustainable. Explain how each resource would impact sustainability.

8. Would it be easy or difficult for someone without a car to live in Greenpoint? Why?

9. Do you think the community where you live is sustainable? If yes, what features make it sustainable? If no, what features would improve your community?

Bonus

The midpoint between the high school and the history museum is (2, 9). What are the coordinates of the history museum?
4. See graph.
5. See table.
6. Accept any answer that is well articulated.
7. Many answers are possible. Some resources that could contribute to sustainability include public transportation systems, a recycling center, businesses that employ many people, and grocery stores within walking distance of senior centers and other groups that may not be able to drive.
8. It would be difficult to walk many of the distances. However, one might be able to reach these distances by bicycle, carpool, or bus.
9. Answers will vary.

**Bonus**
\[
\frac{4 + x_2}{2} = 2; \quad x_2 = 0 \quad \frac{4 + y_2}{2} = 9; \quad y_2 = 14
\]
The history museum is located at (0, 14).

---

ANSWERS

1. **Greenpoint, USA**

2. \[\left(\frac{6 + 4}{2}, \frac{7 + 4}{2}\right) = (5, 5.5)\]

3. Accept any well-explained answer. A community may want to locate certain resources so that they are equidistant from 2 different neighborhoods or community resources. For example, a public library might be located at the midpoint between 2 schools so that both may benefit equally.
The **distance formula** is used to find the distance between 2 separate points, \((x_1, y_1)\) and \((x_2, y_2)\).

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

The **midpoint formula** is used to find the midpoint of a line segment between 2 points, \((x_1, y_1)\) and \((x_2, y_2)\).

\[ m = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]

1. In Mini City, the school location is at \((1, 7)\) and the apartment building location is at \((4, 1)\). Using the distance formula, find the distance between the school and the apartment building.

2. Builders want to create a community center in between the school and the apartment building. What is the midpoint between the 2 locations?

3. Do you think this is a good location for the community center? Why or why not?

4. The founders of Mini City wanted important community buildings to be built an equal distance from Town Hall, the center of the community. Are the post office and the library equidistant from Town Hall?

**ANSWERS**

1. \[ d = \sqrt{(4 - 1)^2 + (1 - 7)^2} = 3\sqrt{5} \approx 6.708 \]

2. \[ \left( \frac{1 + 4}{2}, \frac{7 + 1}{2} \right) = (2.5, 4) \]

3. Possible answers: Yes, it is equidistant to different groups. No, it should be closer to the apartment building since schools already have many resources.

4. No: distance from post office to Town Hall
   \[ = \sqrt{(-2 - 1)^2 + (-3 - 1)^2} \approx 3.606, \]
   and distance from library to Town Hall
   \[ = \sqrt{(3 - 1)^2 + (3 - 1)^2} = 4.472 \]
Data Analysis

Quality of Life

In this 2-day lesson, students develop indicators to measure quality of life and conduct a survey of peers and adults to obtain data for their indicators. They analyze the survey data by calculating measures of central tendency and producing charts to demonstrate their results.
Critical Thinking Questions

• What determines quality of life and happiness?
• How is quality of life measured?
• How does the concept of what is necessary for a high quality of life change over the course of our lives?

Objectives

• Develop quality of life indicators
• Administer a quality of life survey
• Organize data using a box-and-whisker plot
• Analyze data by calculating measures of central tendency
• Compare and contrast student and adult survey responses

Key Concepts

• Box-and-whisker plot
• Mean, median, mode
• Quality of life

NCTM Standards and Expectations Addressed

Data Analysis and Probability: Formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population

Data Analysis and Probability: Find, use, and interpret measures of center and spread, including mean and interquartile range

Data Analysis and Probability: Discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatterplots

Data Analysis and Probability: Use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken

Problem Solving: Solve problems that arise in mathematics and in other contexts

Communication: Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Connections: Recognize and apply mathematics in contexts outside of mathematics

Materials/Preparation—Day 1

Clear a large space where students can stand in a circle

Student handout: Quality of Life Survey Instructions, 1 per group of 2-3 students

Student handout: Quality of Life Survey, 2 per student

Materials/Preparation—Day 2

Student handout: The Good Life, 1 per group

Teacher master: The Good Life

Graphing paper, 1 sheet per group

(Optional) Microsoft Excel may be used to record survey data, determine averages, and graph results

Background and Practice

Background reading: What Do You Need?

Practice worksheet: Practice with Data Analysis
Teacher Instructions—Day 1

1. In a cleared space, ask all students to stand in a circle. Tell them that you will be reading several statements, 1 at a time. Students who agree with a particular statement should take a step inside the circle. Those who disagree with a statement should remain in the outer circle. After each statement is read and students have responded, all students should return to the outer circle.

2. Read the following statements, 1 at a time, and allow students to respond according to the instructions above:
   • People who have many friends have a good life.
   • People who make more than enough money to pay their monthly bills have a good life.
   • People who graduate from college have a good life.

3. Ask the class, “If everyone in the world was ‘living the good life,’ what would we have in common?” Encourage students to think about quality of life as a positive concept, not just a lack of what is negative. For example, the World Health Organization defines “health” as “a state of complete physical, mental and social well-being and not merely the absence of disease or infirmity.”

4. Break the class into groups of 2-3 students and provide each group with 1 Quality of Life Survey Instructions handout. Assign each group 1 of the quality of life categories on the Quality of Life Survey.
   • (Optional) You may have students generate their own quality of life categories.

5. Tell the groups they will have several minutes to read the instructions and develop their quality of life indicators. Circulate through the classroom and assist where necessary.

6. While groups are working, pass out 2 Quality of Life Survey handouts to each student.

7. Have each group report to the class the indicator they came up with. Check that the indicator is something measurable in units of time or quantity, and that a person being surveyed could provide an answer easily. Check that each indicator will fit into the formula: Number of _____ per _____.

8. As groups report their chosen indicators, write them on the board.

9. Each student should write them on his or her 2 blank surveys as well.
10. Each student will administer 1 survey to himself/herself and 1 to an adult to assess their quality of life as defined by these indicators. They will share their results with the class during Day 2 of this lesson.

11. Tell the students to be aware of problems they may encounter when conducting their surveys that could make the data they collect less accurate. Typical issues to be aware of when conducting a survey include:
   - Do people understand the questions?
   - Do people have enough information to give an accurate answer?
   - Are the people they are surveying being honest?
   - Are they surveying people in groups, instead of individually? (People tend to adjust their answers based on what they hear their peers saying.)

12. Conclude the class with a discussion using the following questions.

Discussion Questions—Day 1

1. Can you think of other indicators for the categories? (Note: Do not change the original indicators given by the groups, as you will use those indicators for the survey portion of the exercise. Be sure that the students do not attack each other’s ideas; explain that there are many different ways to measure quality of life.)

2. How do you think people might adjust their lives to be in line with 1 or more of these indicators? (For example, if it was socially accepted that a quality of life measurement for Relaxation is the number of vacation days taken annually, then people might adjust their balance between work and vacation time.)

3. What are some reasons why inaccurate data could be harmful if it is used to make important decisions? How can you ensure that you get useful data from the surveys?

4. The average number of hours worked by people in the United States per year increased from about 33 hours per week in 1979 to about 36 hours per week in 2000.¹ Why do you think people work more hours now than they did 20 years ago? How might an increase in working hours affect quality of life?
Teacher Instructions—Day 2

1. Prepare a blank table on the board or overhead where student groups will share their data. It might look like the following table:

<table>
<thead>
<tr>
<th>Quality of Life Category</th>
<th>Student Results</th>
<th>Adult Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Family</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Friends</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Health</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rest/Relaxation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recreation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Creative Pursuits</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spiritual Pursuits</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Work/Earn Money</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volunteer/Help Others</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The Environment</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Ask the students how their surveying went and if they think the data they collected are accurate.

3. Have each person write the numbers from their 2 completed surveys in the corresponding cells of the table you prepared on the board or overhead. Have each person write a comma after the numbers they report to differentiate their answers from others.

4. Ask them what kinds of calculations they might perform on the data in order to summarize it.

5. Divide the class into the same groups as the previous day (2-3 students each).

6. Pass out a copy of the worksheet, The Good Life, to each group. Tell them to complete the worksheet using the compiled class data from the surveys.

7. Reserve a few minutes at the end of class to discuss the following questions.

Discussion Questions—Day 2

1. Do you think this process accurately measures quality of life? What worked and what was difficult about the process?

2. What was surprising about the results?

3. What could people do differently to change or improve their quality of life?

4. If everyone on the planet measured well-being by these indicators, what would the impact be on the environment, the economy, and society? How is quality of life related to the ability of future generations to meet their needs?
Extension Ideas

1. Graph mean values for each quality of life category by creating a double bar graph, where one series is student data and the other is adult data.

2. Investigate traditional and alternative indicators of quality of life. Two commonly used indicators of quality of life for a group of people are Gross Domestic Product (GDP) or Gross National Income (GNI). Other indicators might include life expectancy or literacy rate. Alternative indicators include the Genuine Progress Index and the United Nations Human Development Index. Determine whether you think overall quality of life in the United States would appear higher or lower using alternative measures of progress and well-being.

Additional Resources


- www.who.int/substance_abuse/research_tools/whoqolbref/en/—The World Health Organization’s Quality of Life project uses a life assessment instrument to measure 26 broad areas, including physical health, psychological health, social relationships, and environment.

Action Project

Have students write a Quality of Life Report for the school newspaper based on results from the survey they completed in class. They can include research, results in the forms of statistics and graphs, and recommendations for ways that people can improve their quality of life.
Quality of Life Survey Instructions

Group Members: ____________________________________________

1. You are going to develop indicators (measurements) to evaluate quality of life based on several categories. You will develop and administer a survey to peers and adults asking for data on the indicators you develop.

2. In the blank below these instructions, write the quality of life category for which your group will develop an indicator.

3. The indicators must be measurable in units of time or quantities and should fit into the formula Number of _____ per ___. For example, if an important element of quality of life is Relaxation, how would you measure it? (e.g., the number of hours per week you do after-school activities or the number of days you take a vacation per year)

4. Make sure your indicators are designed so that a higher number represents an increase in the quality of life category. For example, the indicator “number of hours you sleep a night” would yield higher numbers for a better quality of life. However, the indicator, “number of times you yawn in a day,” would yield higher numbers for a lower quality of life.

5. Consider how easy or difficult it will be for the people you survey to provide data for the indicators. For example, an indicator of Recreation could be the number of milliseconds a person spends playing sports every day, but not many people can tell you how many milliseconds they spend doing anything!

6. Write your indicator below.

7. When all groups are finished developing their indicators, each group will report their chosen indicator to the class. Write these indicators on your 2 blank surveys.

8. Administer 1 survey to an adult and 1 to yourself.

Your assigned Quality of Life category: ____________________________

Your indicator: Number of ____________ per ______________
# Quality of Life Survey

Survey administered by (your name): ________________________________

Person being surveyed is: □ Student (Age 18 or younger)  □ Adult (Older than 18)

<table>
<thead>
<tr>
<th>Quality of Life Category</th>
<th>Indicator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Family</td>
<td>Number of ________________________ per ______</td>
</tr>
<tr>
<td></td>
<td>Answer:</td>
</tr>
<tr>
<td>Friends</td>
<td>Number of ________________________ per ______</td>
</tr>
<tr>
<td></td>
<td>Answer:</td>
</tr>
<tr>
<td>Health</td>
<td>Number of ________________________ per ______</td>
</tr>
<tr>
<td></td>
<td>Answer:</td>
</tr>
<tr>
<td>Rest/Relaxation</td>
<td>Number of ________________________ per ______</td>
</tr>
<tr>
<td></td>
<td>Answer:</td>
</tr>
<tr>
<td>Recreation</td>
<td>Number of ________________________ per ______</td>
</tr>
<tr>
<td></td>
<td>Answer:</td>
</tr>
<tr>
<td>Creative Pursuits</td>
<td>Number of ________________________ per ______</td>
</tr>
<tr>
<td></td>
<td>Answer:</td>
</tr>
<tr>
<td>Spiritual Pursuits</td>
<td>Number of ________________________ per ______</td>
</tr>
<tr>
<td></td>
<td>Answer:</td>
</tr>
<tr>
<td>Work/Earn Money</td>
<td>Number of ________________________ per ______</td>
</tr>
<tr>
<td></td>
<td>Answer:</td>
</tr>
<tr>
<td>Volunteer/Help Others</td>
<td>Number of ________________________ per ______</td>
</tr>
<tr>
<td></td>
<td>Answer:</td>
</tr>
<tr>
<td>The Environment</td>
<td>Number of ________________________ per ______</td>
</tr>
<tr>
<td></td>
<td>Answer:</td>
</tr>
</tbody>
</table>
Investigations

1. Copy the compiled survey results into the following table:

<table>
<thead>
<tr>
<th>Quality of Life Category</th>
<th>Student Results</th>
<th>Adult Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Family</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Friends</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Health</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rest/Relaxation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recreation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Creative Pursuits</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spiritual Pursuits</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Work/Earn Money</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volunteer/Help Others</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The Environment</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Find the mean for the Student Results for the Family indicator.

3. Find the mean for the Adult Results for the Family indicator.

4. Find the median of the Student Results for the Health indicator.

5. Organize the data for the Student Results for the Health indicator in a box-and-whisker plot.

6. Write an expression representing the results in the top 25% for Student Results for the Health indicator.

7. Find the mode for the Student Results for the Recreation indicator.

8. Find the mode for the Adult Results for the Recreation indicator.

9. Based on the survey indicators, which group appears to have a higher quality of life: students or adults? Name at least 1 factor that might contribute to this group’s higher scores on your survey.

ANSWERS

1. Answers will vary.

2. Add all Student Results for Family, then divide by the number of responses.

3. Add all Adult Results for Family, then divide by the number of responses.

4. This is the “middle” number of the dataset of Student Results for Health. If the total number of data points is odd, the median will be the middle number of the dataset when it is ordered from smallest to largest. If the total number of data points is even, find the average of the 2 numbers in the middle of the dataset.

5. As an example, if the third quartile (separating the bottom 75% from the top 25%) is 30, a possible expression would be \( x > 30 \).

6. The mode is the number that appears most frequently in a set of data. More than 1 number may be a mode.

7. The mode is the number that appears most frequently in a set of data. More than 1 number may be a mode.

8. Answer depends on survey data.
Practice with Data Analysis

Review the above graphs of 2007 American Time Use Survey data.¹

1. Why is it difficult to directly compare the 2 graphs shown?

2. How would you respond to someone who asked you how males and females spend their time differently?

3. A group of 9 students were asked the question, “How many hours of TV do you watch each week?” Find the median of their responses: 14, 15, 13, 14, 26, 11.5, 16, 15, 12

4. What is the mean number of hours of TV watched by the 9 students above? Round your answer to 2 decimal places (the hundredths place).

5. Which measure do you think more accurately represents the data shown: mean or median? Explain your answer.

6. How would the mean be affected if you removed the student response of 26 hours?

ANSWERS

1. They have different y-axis scales
2. Men work more outside the home and watch more TV. Women spend more time doing housework, caring for family, and volunteering.
3. 14
4. ≈ 15.17 hours
5. Median—more than half of the responses are 14 or less, whereas the mean is above 15 and only 2 data points are greater than 15
6. It would be reduced (to approximately 13.8 hours).
Integers:  
Climate Change & Sea Level Rise

One way to include real world examples in a study of integers is to use data related to land elevations. Climate change is a topic of much interest, and sea levels around the world are one aspect of climate change. Students can investigate how sea level rise will affect elevations in different locations.

Datasets


<table>
<thead>
<tr>
<th>Location</th>
<th>Land Elevation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead Sea (between Jordan and Israel)</td>
<td>−413 m</td>
</tr>
<tr>
<td>Death Valley, California</td>
<td>−86 m</td>
</tr>
<tr>
<td>Manhattan, New York</td>
<td>0 m</td>
</tr>
<tr>
<td>Biloxi, Mississippi</td>
<td>5 m</td>
</tr>
<tr>
<td>Mt. Everest (between Tibet and Nepal)</td>
<td>8850 m</td>
</tr>
</tbody>
</table>

Teaching Ideas

1. Start with a review of integers on a number line. Students should be familiar with addition and subtraction of integers.

2. Use the concept of a number line to illustrate sea level. By using a vertical number line, students can visualize land elevations relative to sea level. Positive numbers correspond to land elevations above sea level. Negative numbers correspond to land that is below sea level.

3. To reinforce the idea that distance is always positive, have students determine the absolute values of a variety of elevations above and below sea level.

4. During the past century, average sea level rise was approximately 2 mm per year. Challenge students to determine how particular locations will be affected by continued sea level rise. Students can also find out the elevation where they live and calculate how this elevation will be affected 50 or 100 years from now if sea level rises approximately 2 mm/year.

5. Ask students to consider how sea level rise will impact people around the world:
   - How might rising sea levels connect to worldwide human migration?
   - In the United States, how will rising sea levels affect cities along the East Coast and the Gulf Coast?

Additional Resources

- [www.geo.arizona.edu/dgesl/research/other/climate_change_and_sea_level/sea_level_rise/sea_level_rise_old.htm#images](http://www.geo.arizona.edu/dgesl/research/other/climate_change_and_sea_level/sea_level_rise/sea_level_rise_old.htm#images) — The NASA Jet Propulsion Laboratory has created an image that illustrates worldwide changes in sea level from 1993 to 2008. An accompanying caption explains causes and impacts of sea level rise.

- [www.geo.arizona.edu/dgesl/research/other/climate_change_and_sea_level/sea_level_rise/sea_level_rise_old.htm#images](http://www.geo.arizona.edu/dgesl/research/other/climate_change_and_sea_level/sea_level_rise/sea_level_rise_old.htm#images) — View maps illustrating which parts of the world are most susceptible to sea level rise. Use one of the North America maps to estimate the percentage of land area that may be submerged below sea level for each meter of sea level rise.

Information is often presented in the form of indices or rankings. For example, movie ratings are reported as an index. Movies are ranked according to a particular scale (i.e., 5 stars). Information related to international governance is also often reported as indices. Students can examine different factors related to governance to determine whether correlations exist between governance (in this case, the type of government) and other country-specific variables.

**Datasets**

Datasets from the World Resources Institute provide several different indices with country-specific values that students can use to create scatterplots. Just a few indices include: Digital Access Index, Freedom Index, Press Freedom Index, and Corruption Perceptions Index. To access these numbers, visit [www.earthtrends.wri.org](http://www.earthtrends.wri.org) and click on Environmental Governance and Institutions, then Data Tables.

Below is a sample of data provided by the World Resources Institute. Level of Democracy is a measure of the degree to which a country is democratic (on a scale of −10 to 10, where −10 is strongly autocratic and 10 is strongly democratic). The Digital Access Index measures the level of access people in a given country have to Internet and digital information (on a scale of 0 to 100, where 0 is no access to digital information and 100 is full access to digital information).

<table>
<thead>
<tr>
<th>Country</th>
<th>Level of Democracy</th>
<th>Digital Access Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kazakhstan</td>
<td>−6</td>
<td>41</td>
</tr>
<tr>
<td>Angola</td>
<td>−3</td>
<td>11</td>
</tr>
<tr>
<td>Côte d’Ivoire</td>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>Argentina</td>
<td>8</td>
<td>53</td>
</tr>
<tr>
<td>United States</td>
<td>10</td>
<td>78</td>
</tr>
</tbody>
</table>

**Teaching Ideas**

1. Pass out index cards to students and have them write a word that comes to mind when they hear the word “democracy.” Then illustrate a democratic process by having the class vote on their favorite word associated with democracy.

2. Review prior student knowledge about democracies. *In a democracy, citizens, rather than ruling authorities, make decisions that guide their country.*

3. Introduce the concept of an index and share different examples (e.g., movie ratings, heat index, Dow Jones Index). Expand upon this idea to explain Level of Democracy and Digital Access Index numbers.

4. Ask students to consider how the level of democracy may be related to the availability of news and information in a country.

5. Use World Resources Institute data to create scatterplots and construct lines of best fit to determine whether level of democracy is correlated with access to information.

6. Encourage students to think critically about democracy and freedom of information:
   - Describe what you think a country with a digital access of 100 would be like. Would it necessarily be a democracy?
   - Do you think in some cases restrictions on our freedoms are good?

**Additional Resources**


- [www.usaid.gov](http://www.usaid.gov)—The United States Agency for International Development (USAID) provides data related to democracy and governance for the world’s major regions.
One idea for studying real data and polynomials is to have students analyze actual survey results. They can analyze survey responses from different populations by using measures of central tendency. Having students make recommendations to improve survey questions and to gather more accurate information requires critical thinking.

Datasets
Survey results related to student environmental knowledge and perceptions can be found at www.hamilton.edu/levitt/Sustainability/youth_poll_data.html. In 2006 a group from Hamilton College administered a short quiz to 900 high school students in order to gauge their knowledge about climate change. Students were also asked about their environmental attitudes, pro-environment behaviors, and policy preferences. The following tables show a sample of the findings from the report, “Climate Change and Environmental Issues Poll.”

<table>
<thead>
<tr>
<th>Number of Students</th>
<th>Mean Score on Climate Change Quiz (reported as a percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>357</td>
</tr>
<tr>
<td>Female</td>
<td>543</td>
</tr>
<tr>
<td>Interested in politics</td>
<td>154</td>
</tr>
<tr>
<td>Not interested in politics</td>
<td>744</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Strongly Agree</th>
<th>Mildly Agree</th>
<th>Mildly Disagree</th>
<th>Strongly Disagree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Humans have the right to modify the natural environment</td>
<td>24.94%</td>
<td>41.61%</td>
<td>23.04%</td>
</tr>
<tr>
<td>The earth is like a spaceship with only limited room and resources</td>
<td>46.59%</td>
<td>30.84%</td>
<td>12.96%</td>
</tr>
</tbody>
</table>

Teaching Ideas
1. Have students answer the survey questions from the Hamilton College report.
2. Tally correct and incorrect responses to the questions related to knowledge about climate change (Hamilton report, Appendix B, 18-26).
3. Students can calculate the mean, median, and mode of the class’s scores and compare their results to data from Hamilton College.
4. Connect ideas from the survey to polynomials. Consider this hypothetical problem:
   • Studying climate change at school over several school years should improve a person’s score on a climate change quiz. Use the following formula to determine the expected quiz score for a person who has learned about climate change during 3 different school years and who watches an average of 2 hours of TV news per week: \(2b(ab + 3b)\), where \(a\) = average number of hours per week spent watching TV news, and \(b\) = average number of months spent learning about climate change at school per year.
5. Lead a student discussion using the following questions:
   • What would you change about the survey to better assess climate change knowledge?
   • Why do you think the students who took the Hamilton College survey in late 2006 did not know more about climate change?
   • Is climate change an important issue to learn about?

Additional Resources
• www.ipcc.ch—The Intergovernmental Panel on Climate Change (IPCC) provides much information on past, current, and predicted future shifts in climate change.
• www.epa.gov/climatechange/kids/index.html—The U.S. Environmental Protection Agency offers a website for students that explains different components of climate change.
Lesson 1. Number Patterns: Waste & Recycling


Lesson Handout: Waste Not, Want Not


4. “Fun Facts About PET.”


6. Ibid.

Lesson 2. Introduction to Algebra: Poverty & Microcredit


Lesson Handout: Microcredit Business Application


Lesson Handout: The Ups and Downs of Population


2. CIA, The World Factbook.

3. Ibid.

Lesson 4. Solving Algebraic Equations: Personal Health Choices


Lesson Handout: You Are What You Eat
1 All caloric value and daily values were derived from www.caloriecount.about.com.

Practice Worksheet: Practice with Solving Equations

Lesson 5. Data & Graphs: Youth Conflict
Lesson Handout: Increasing the Peace

Practice Worksheet: Practice with Data & Graphs

Lesson 6. Number Theory: Consumption Choices

Lesson Handout: Paper or Plastic?

Practice Worksheet: Practice with Fractions

Lesson 7. Rational Numbers: Financial Decisions

Practice Worksheet: Practice with Percent

Lesson 8. Proportion, Percent, & Probability: Global Health
2 Estimate based on CIA, The World Factbook.
Lesson Handout: *Live Long and Prosper*


Practice Worksheet: *Practice with Rates, Ratios, & Proportions*


Lesson 9. Solving Inequalities: Carbon Emissions

Lesson Handout: *Budgeting for Climate Change*

1. Target numbers are based on the Kyoto Protocol target of 1,252 million metric tons C and a current U.S. population of 305,000,000.


*** The PB&J Campaign, [www.pbjcampaign.org/numbers](http://www.pbjcampaign.org/numbers).

**** EPA Individual Emissions Calculator, [www.epa.gov/climatechange/emissions/individual.html](http://www.epa.gov/climatechange/emissions/individual.html).


Practice Worksheet: *Practice with Inequalities*


Lesson 10. Spatial Thinking: Solar Power


Lesson Handout: *What’s Your Angle?*


Practice Worksheet: *Practice with Angles*

Lesson 11. Area & Transformations: Wildlife Habitats

Lesson Handout: Home Sweet Habitat


Lesson 12. Surface Area & Volume: Sustainable Design


2 Ibid.

Lesson 13. Linear Functions: Global Education


Lesson Handout: Making the Grade


Lesson 15. Data Analysis: Quality of Life


Practice Worksheet: Practice with Data Analysis

Thank you to the generous and talented photographers who contributed photos.

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Seattle Center Fountain by Jessica C Levine
Zebras by Kim Rakow Bernier
Students working together by Craig Snell

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Lesson 4. Solving Algebraic Equations: Food Choices
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p. 48- Peruvian girls by Sharla Halvorson
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p. 49- Gentle and Violent shoe art by Laura Skelton

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p. 63- U.S. Capitol by Chris Creamer
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p. 70- Health Center at Gunhill by Mario Burger
p. 71- Pharmacy in Japan by Sherry Deckman
p. 72- Multigenerational family by Elizabeth Benedict Huttman

Lesson 9. Solving Inequalities: Carbon Emissions
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p. 81- Bike parking post by Laura Skelton
p. 82- Smokestacks and cooling towers by PhotoDisc, Inc.

Lesson 10. Spatial Thinking: Solar Power
p. 87- Volunteers installing solar panels on roof by Kate Davison, Greenpeace Southeast Asia
p. 88- Cooling towers by PhotoDisc, Inc.
p. 89- Solar panels by Rodd Halstead
p. 90- Wind turbine by PhotoDisc, Inc.
p. 91- Windmill by Eddie Martinez
p. 91- Wind farm by PhotoDisc, Inc.
p. 95- Houses with solar panels illustration by DECODE, Inc.
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p. 98-  Bald eagle by PhotoDisc, Inc.
p. 99-  Orangutan by PhotoDisc, Inc.
p. 100- Snow leopard in zoo by Heidi Connal
p. 100- Mountains in Pakistan photo by Bill Hogue, Courtesy of Snow Leopard Trust
p. 100- Ajar Canyon in Afghanistan by Abraham J Sheppard
p. 101- Wild goat ©iStockphoto/sirius_r
p. 101- Snow leopard by PhotoDisc, Inc.

Lesson 12. Surface Area & Volume: Sustainable Design
p. 103- Wall of windows at Seattle Public Library by DECODE, Inc.
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p. 105- Bamboo Shapes photo courtesy of Horst Bierau (foto.bierau.net)
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Lesson 13. Linear Functions: Systems & Global Education
p. 112- Teacher with students ©iStockphoto/track5
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p. 114- Young children in Uganda by Kim Rakow Bernier
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p. 117- Students with teacher in classroom in Ecuador ©John and Lisa Merrill
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Lesson 14. Midpoint & Distance Formulas: Resource Distribution
p. 122- Seattle Center Fountain by Jessica C Levine
p. 123- On the Double by Jessica C Levine
p. 124- Hong Kong neighborhood by DECODE, Inc.
p. 125- Model neighborhood by Casey Broadwater

Lesson 15. Data Analysis: Quality of Life
p. 129- Smiling couple by Laura Skelton
p. 130- Rollercoaster by Eddie Martinez
p. 131- Men playing dominoes by Sheeba Jacob
p. 132- Woman scuba diving by PhotoDisc, Inc.
p. 133- Man and child at rally in Washington, D.C. by Brian Greer
p. 134- Step It Up rally by Jesse Stanley
Real World Math: Engaging Students through Global Issues

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- Modeling integers
- Algebraic equations
- Data & graphs
- Number theory
- Rational numbers
- Proportion, percent, & probability
- Solving inequalities
- Spatial thinking
- Area & transformations
- Surface area & volume
- Linear functions
- Midpoint & distance formulas
- Data analysis

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- Collaboration
- Hands-on activities
- Action project ideas
- Bonus and extension problems

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