Winter 2021

The Graph Menagerie: An exploration of the intersection of math, biology, and art

Maggie Barry
Western Washington University

The Graph Menagerie

An exploration of the intersection
of math, biology, and art

By Maggie Barry

Advised by Dr. Jeanine Amacher

Abstract

This project explores interdisciplinarity with a focus on how math and biology can interact with art. My main objective was to create art by graphing the silhouettes of animals. I selected ten animals from a variety of classes and habitats and used a collection of equation types such as linear, quadratic, trigonometric, and circular to draw an outline of each animal. I performed stretches, compressions, and shifts to control the size and position of each equation and set domains and ranges to determine how much of each line was visible on the graph. In the first section of this paper, I present my methods and collection of these graphs. To supplement this, in the second section I examine a range of artists with ties to biology and math. This includes historical examples such as Leonardo da Vinci and modern examples such as David Goodsell. This exploration shows that there is a variety of ways for art, biology, and math to interact and there are many meaningful outcomes of this interdisciplinarity. In addition, art can be made accessible to anyone by approaching it through their interests.

Table of Contents

I. The Graphs

Introduction

I am a biology/math major with minors in chemistry and honors, which has allowed me to learn from many perspectives and see connections between subjects that I otherwise would have missed. This has also led me to be very STEM-oriented in my ways of thinking and seeing the world. Through the Honors Program, I branched out into literature and history more than I ever would have within my major, but one field that I remain distant from is art. Even in high school, I was only involved with art in classes to fulfill graduation requirements. For this project, I wanted make art more accessible to me by exploring it through my interests.

I decided to use graphs as my medium because I have had years of math classes learning about functions and ways to manipulate them, and in my time as a tutor in the past couple of years, I've spent even more time thinking conceptually about equations and their transformations. I narrowed the topic to graphing animals because I felt it spoke to the biology half of my major, tying the whole idea together and making the product into a visual representation of the interdisciplinarity that has been so central to my education.

Methods

I began each graph by choosing an animal and finding a photo I could use as a reference to produce an identifiable silhouette. To guide my selection, I tried to pick at least one animal from each of the more well-known classes (mammal, bird, reptile, etc.) and from a variety of habitats. This also created a portfolio showcasing the biodiversity of earth, though in the end it was definitely skewed towards mammals (Fig. 1).

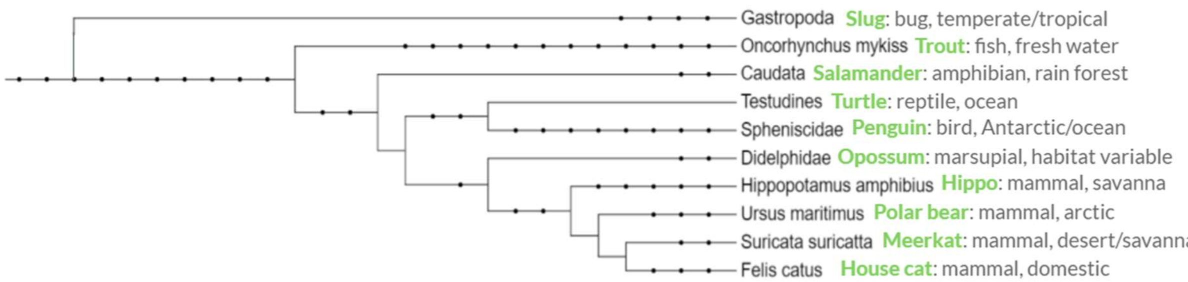

Figure 1: Phylogenetic tree of the ten selected animals with scientific and common names, family or general grouping, and habitat.

I made all my graphs in Desmos, a free online graphing calculator. To start a graph, I would pick a point on the reference picture (usually around the head), focusing on some small piece of it that could be made from a single function (such as the forehead), then think about what equations could be manipulated into that shape. I put that basic equation ($y=x^2$, $x^2+y^2=a$, $y=mx+b$, etc.) into Desmos, then used stretches and shifts (achieved by multiplying and adding constants to the variables) to position it on the graph and shape it to match the picture. For most of the graphs, I tried to set it up such that the end product would be centered near the origin, but it was sometimes hard to predict how the scaling would work out based on the first few equations and the animals often ended up being bigger or smaller than I anticipated.

I was much more exploratory with the earlier graphs in terms of the types of equations I used, including trigonometric, exponential, and rational functions. For later graphs, I mainly used circle, quadratic, and liner equations with good results. I would view this trend as refining my technique and becoming more efficient at recognizing what type of equation will be able to replicate a given segment from the reference picture. However, a great deal of guess and check and minor adjustments were still needed to get each function in the proper position and shape.

In the course of this project, I did learn about a type of equation new to me: a combined quadratic and trigonometric function that makes a wavy parabola which can convey fluffy fur (see Fig. 10 and Appx. I). This was really interesting to find because I am familiar with both of those functions individually and after seeing them together in examples, it made sense that they could be used together to make this type of curve, with the quadratic making the parabolic shape and the cosine function adding the oscillations. When making the cat graph, I first searched for how to write an equation for wavy circles, which required the use of polar coordinates in conjunction with the sine function. This method worked in achieving the right texture, but I could not figure out how or if it was possible to change the center point of the circle to something other than the origin, so I decided to explore different ways to make this type of line and found the parabolic method, which was highly effective.

In terms of connecting the equations to make a cohesive portrait, Desmos tells you the coordinates of the intersection points between two curves, which made naming domains and ranges very simple. To help lines blend together, I tried to adjust neighboring curves to be tangential through their intersection, and if that was not achievable, I would use a small circle function that I could manipulate to be tangential to both lines and bridge the gap between them, then adjust domains and ranges of all three functions, resulting in a smoother overall transition.

Desmos allows you to un-select functions so they don't appear on the graph, but are still saved and available, which allowed me to compare different shapes for certain sections of the graphs to decide which worked best (Fig. 11). Some of the equations I decided not to use are still in my function lists and illustrate how much tinkering is involved in making these graphs (Appx. A-J). I also used this feature to test how many internal details I wanted to include, such as eyes. For the most part, I decided against additions, instead minimizing features that were not part of the silhouette. In some cases, however, it did improve the picture such as the trout's fins and the turtle's shell pattern (Fig. 5 and 6).

There is an option to change the color of the line for each function, which I decided not to use because the multiple colors show the quantity of equations that go into making each picture and how some sections are all one function while others are made of several small pieces of different equations all joined together.

I chose not to overlay a grid on the reference pictures because it would have diminished the challenge if I was merely replicating what was essentially already a graph, just without the equations. Also, I appreciated being able to use my existing knowledge of functions to try to visualize what equation could match the shape of the picture, then modifying it as I went along until it looked right, often looping back to adjust it further when more of the picture was done.

Results

Polar Bear: mammal/ursidae, arctic (Fig. 2, Appx. A)

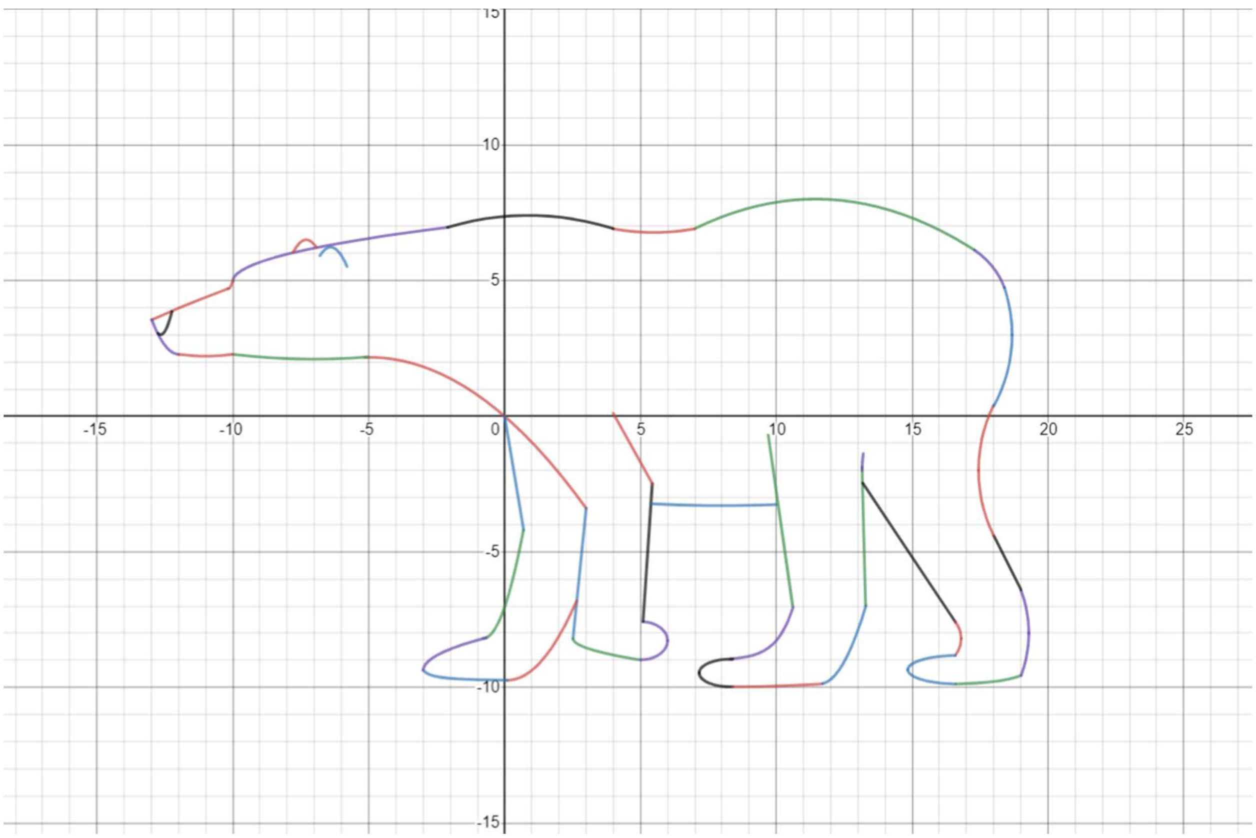

Figure 2: Polar bear graph, made with 40 equations

This was the first graph I made, and it was my test to decide if the project idea was feasible. It gave me good insight into the amount of tinkering required and provided a good review on function transformations. I asked for a lot of second opinions on how to improve the graph to be more recognizably a polar bear. I also decided against giving it a tail because I couldn't get it to look right.

Penguin: bird, Antarctic/ocean (Fig. 3, Appx. B)

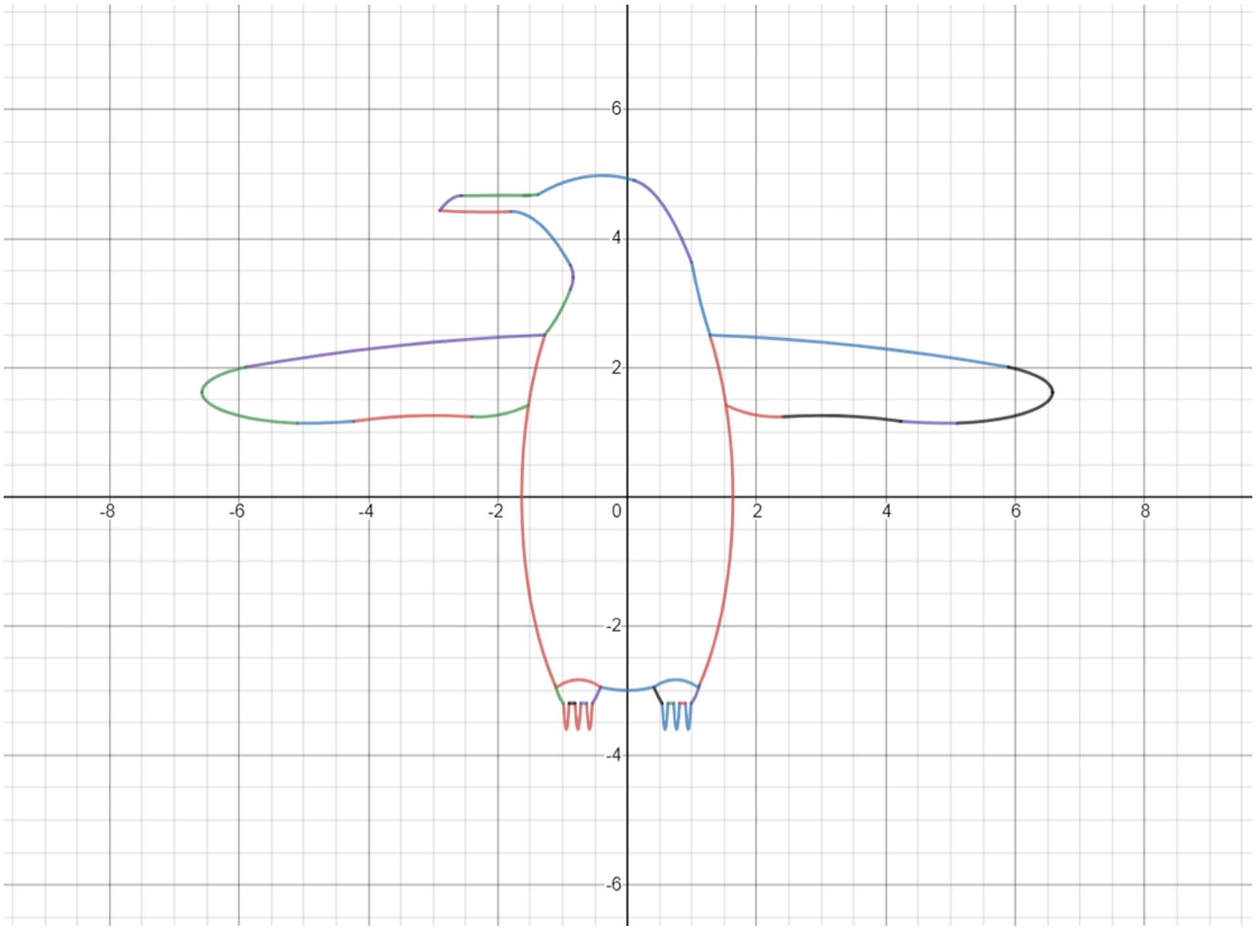

Figure 3: Penguin graph, made with 34 equations

I set the penguin up symmetrically across the y-axis, which made most of the body easier to graph because from the neck down, both sides are made of the same equations, just mirrored (achieved mainly with switching the signs of the horizontal shift and domain). Also, the toes on each foot are made with a single sine graph, which is a really good example of being able to effectively replicate a shape from the reference photo with an equation.

Salamander: amphibian, rain forest (Fig. 4, Appx. C)

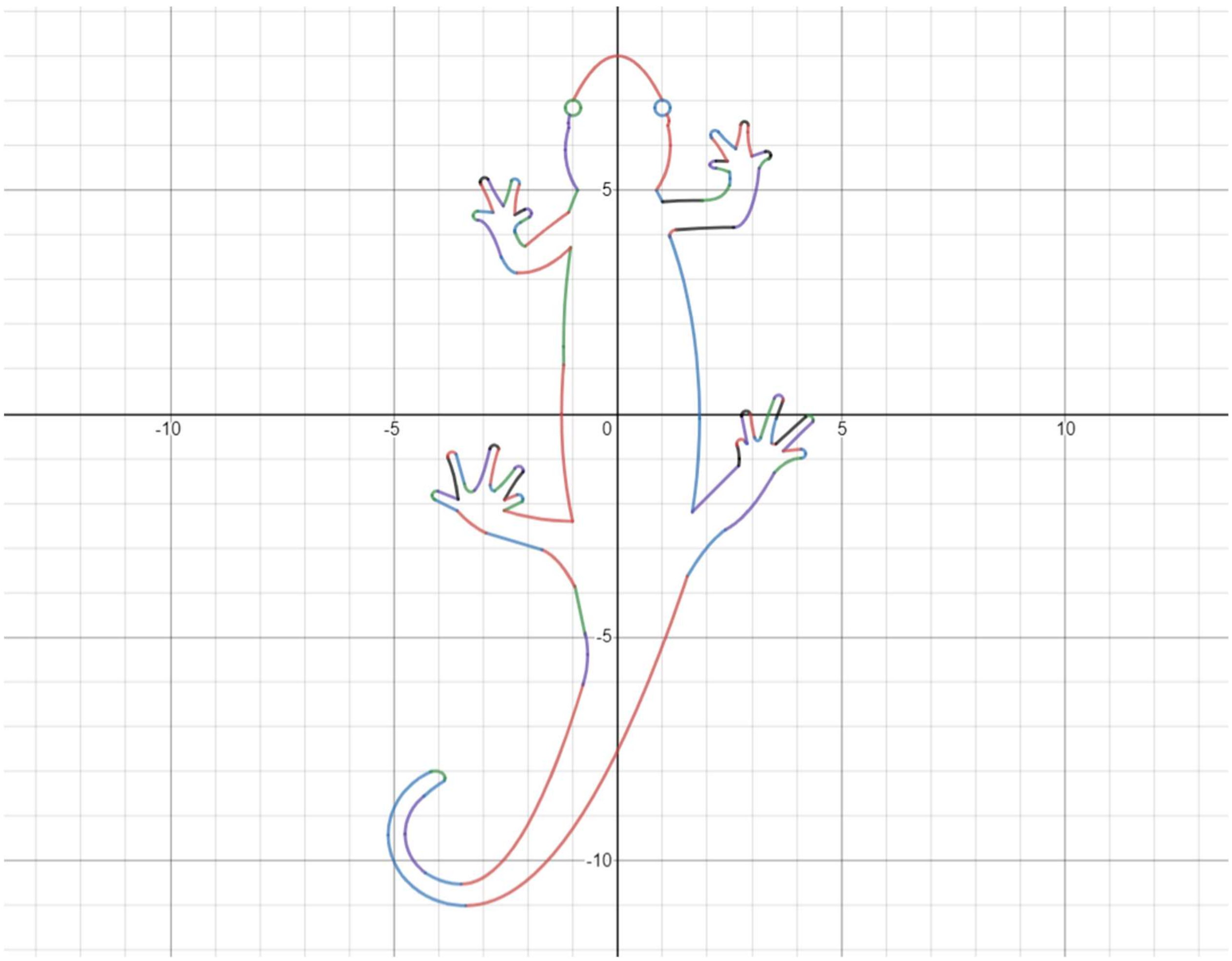

Figure 4: Salamander graph, made with 99 equations

This graph taught me a lesson in terms of assessing the complexity of a silhouette. It took much longer (about 9 hours) and significantly more patience to make than I anticipated because of the toes. After this, I was weary of choosing animals with many small appendages and didn't attempt another until much later.

Trout: fish, fresh water (Fig. 5, Appx. D)

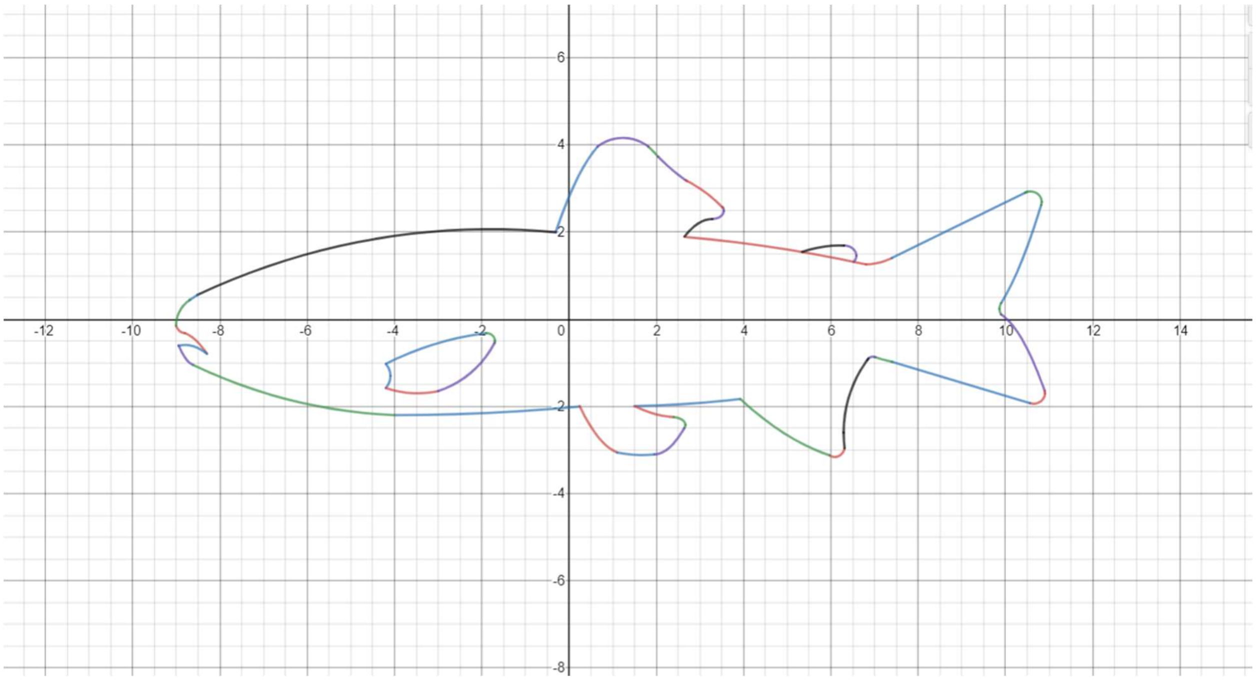

Figure 5: Trout graph, made with 43 equations

I spent a lot of time adjusting the fins of the trout. I looked at many supplementary reference pictures to get a better idea of what their fins look like in different positions to help make the graph look right. This graph also helped me learn that sometimes it's best to keep moving forward to get a full rough draft before revising, rather than fixating on trying to get each segment right before moving on to the next.

Turtle: reptile, ocean (Fig. 6, Appx. E)

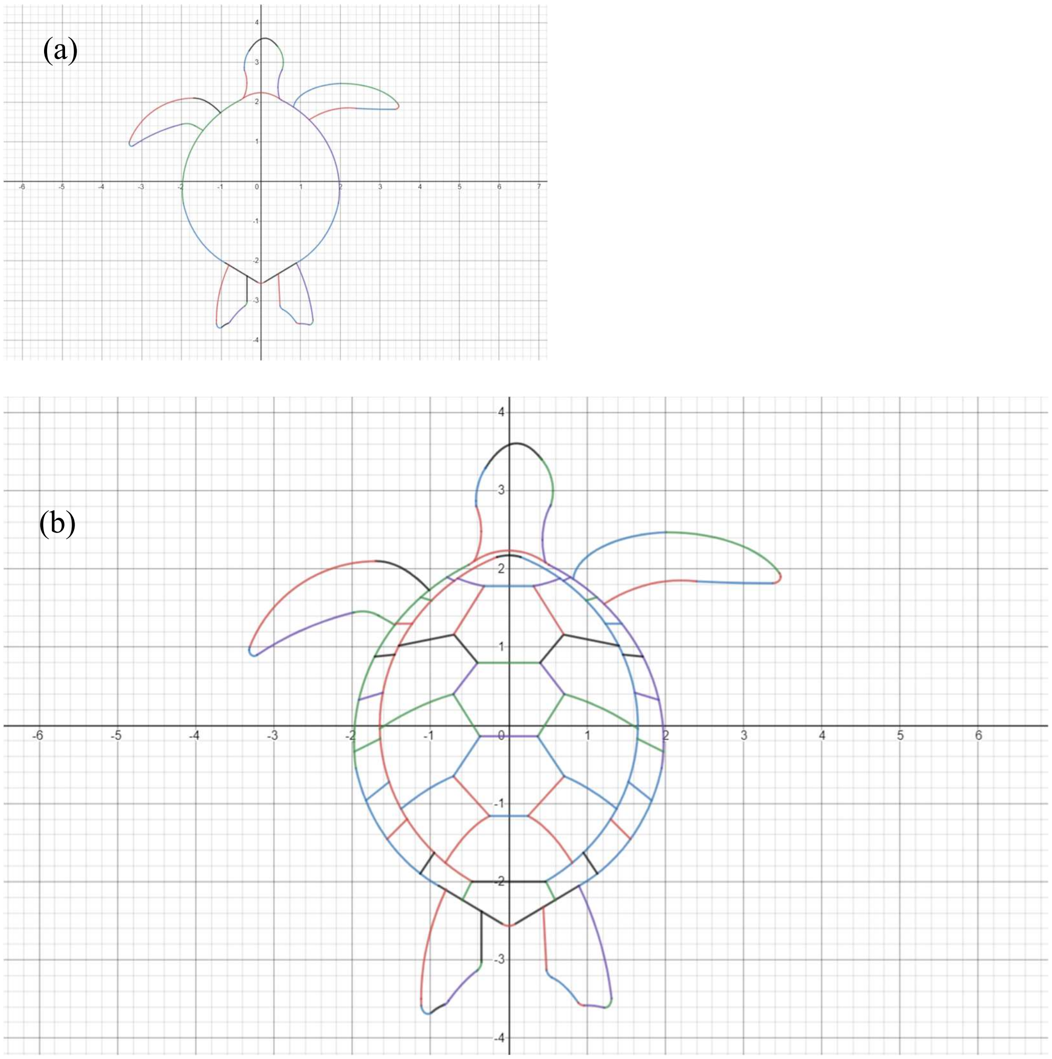

Figure 6: turtle graph (a) before and (b) after adding the shell pattern (66 equations)

A few weeks after completing the turtle's outline, I decided to go back and add the shell pattern, which significantly improved the picture. This graph had symmetry, but in contrast to the penguin, most of the mirrored segments are drawn with a single equation, which I was able to do because the functions making up the shell pattern are also symmetrical across the y-axis. This allowed me to set the range rather than the domain when restricting the lines, so one equation could be present on both sides of the y-axis. In making this, I used several absolute value functions, which I had not played with much beyond algebra class in high school.

Slug: invertebrate/bug, temperate/tropical (Fig. 7, Appx. F)

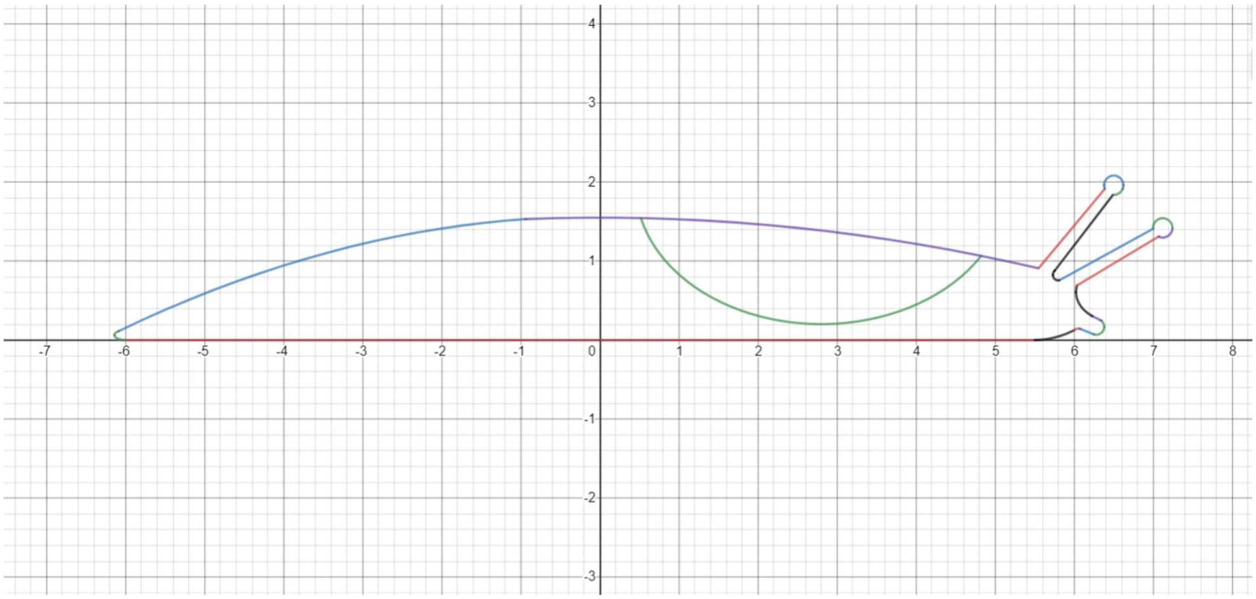

Figure 7: Slug graph, made with 20 equations

This was by far the least complicated and quickest graph to make (it took one hour) and I thoroughly enjoy the end product because it is so simple and so clearly a slug. I didn't particularly want to go too far into the different types of bug-like animals, so I just picked a slug to represent the whole group.

Meerkat: mammal/mongoose, desert/grassland/savanna (Fig. 8, Appx. G)

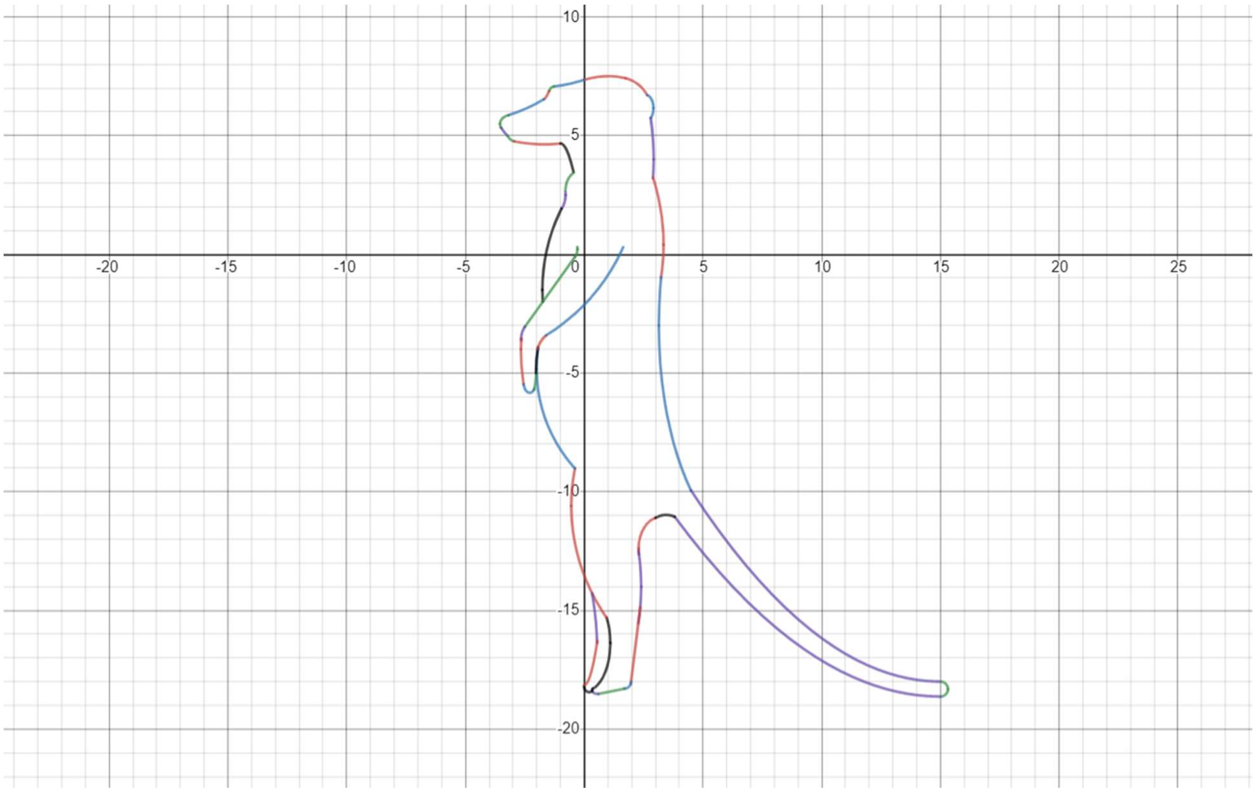

Figure 8: Meerkat graph, made with 43 equations

I decided on a meerkat for an animal from the desert because it has such a distinct outline in the "on watch" position and I wanted to try to replicate that as a graph.

Hippo: mammal/hippopotamidae, savanna/grassland (Fig. 9, Appx. H)

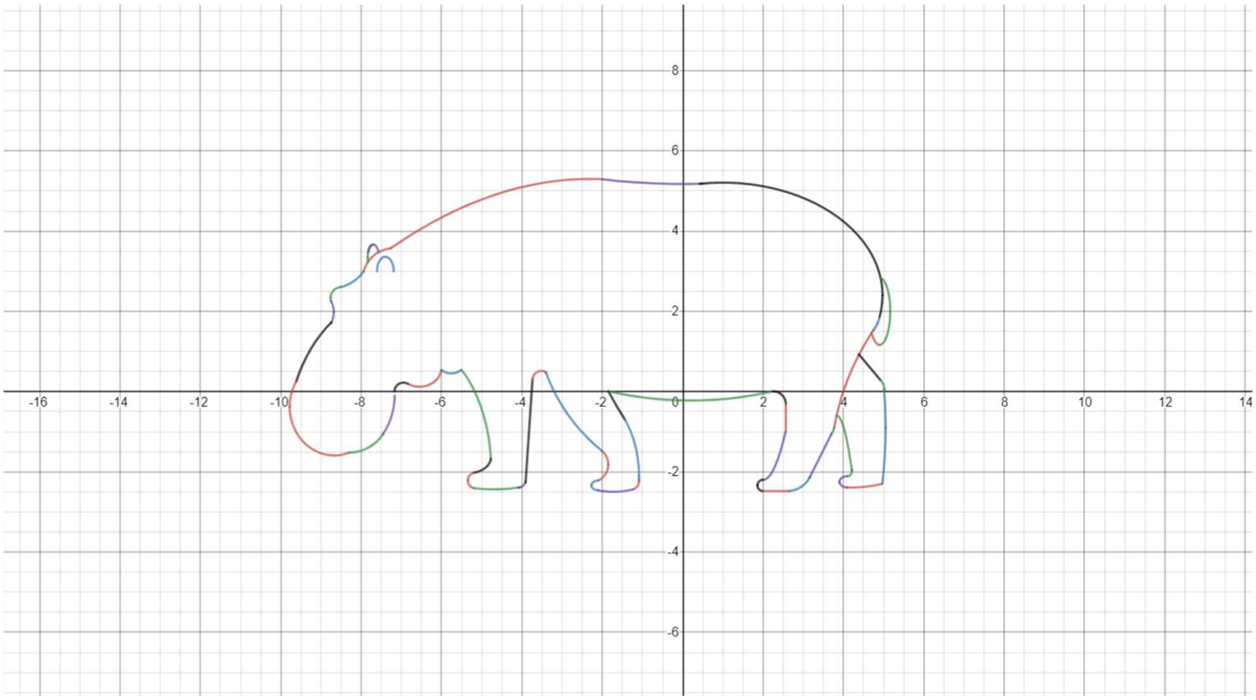

Figure 9: Hippo graph, made with 51 equations

I took a screen recording while making of the hippo and sped it up into a time lapse. It's interesting to watch my graphing process accelerated; there were so many minor equation adjustments, deleting and altering functions, and looping back to change things as I went along.

House cat: mammal/felidae, domestic (Fig. 10, Appx. I)

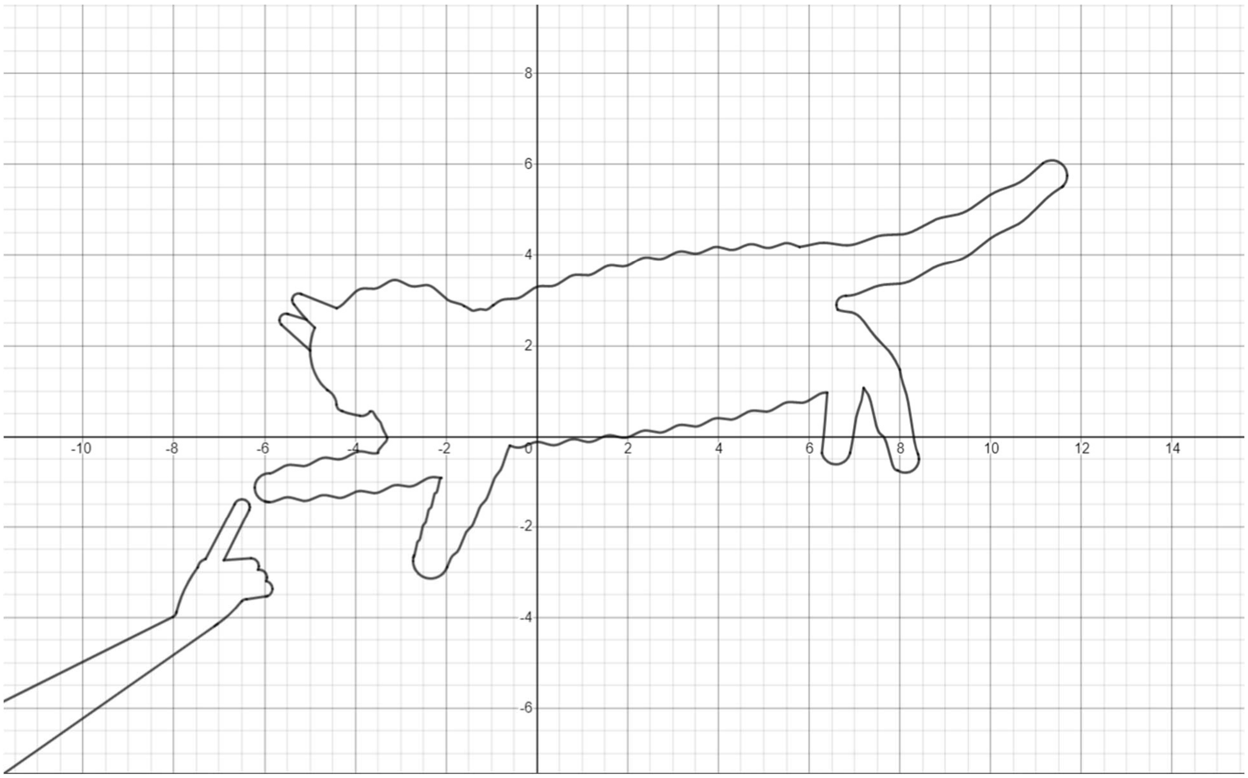

Figure 10: House cat graph, made with 50 equations

This is my roommate's cat, Luka. In making this graph, I learned a new type of function: using a trigonometric equation to make a parabola wavy. This arose from my dissatisfaction at regular parabolas and circles not conveying his fluffiness, so I did some research and found this as the perfect solution. I made all the lines black in contrast to all the other graphs because I printed and framed it as a gift.

Opossum: marsupial, habitat variable within the Americas (Fig. 11, Appx. J)

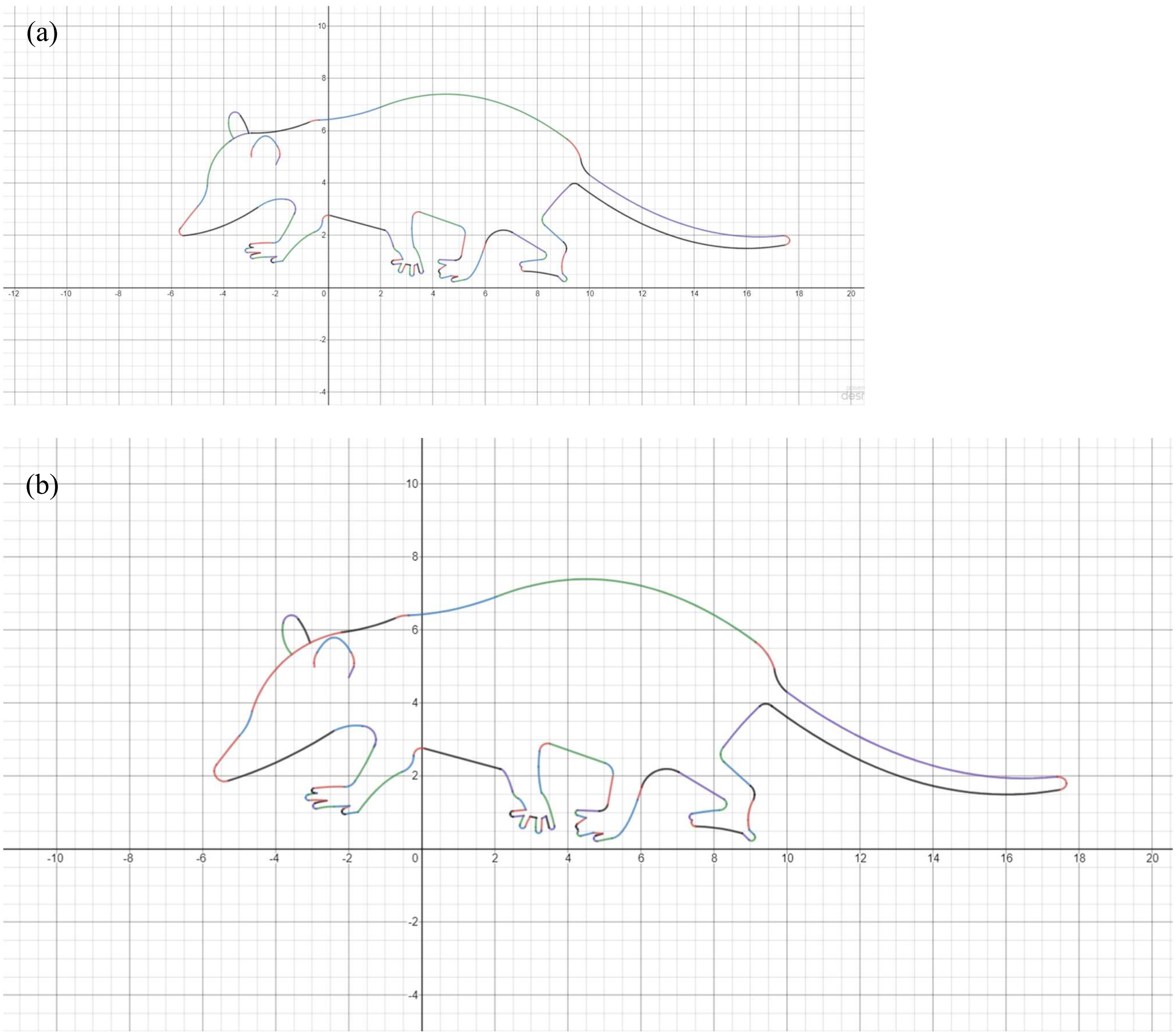

Figure 11: Opossum graph. (a) one of the potential head shapes, (b) the final, made with 94 equations

My grandma suggested an opossum and after a long break from animals with distinct toes, I was able to oblige her. I presented a few different head shapes to people to help me decide which was the most accurate and recognizable.

Discussion

An invaluable lesson I've learned through this process is that showing your pieces to other people and asking for their input is crucial in the formation of a final product. I frequently found myself staring at a graph wondering why it didn't look right when as far as I could tell the lines matched the reference photo well, but asking the people around me for their insight always served to remedy these issues and produce a better picture.

I appreciated how graphing was similar to drawing in the sense that you "sketch" the outlines/general shapes of your picture, then refine them to the final product. I felt that drawing with equations was more in tune with my artistic abilities and patience because I had more control over how the lines were formed and I could fine-tune them more accurately than I would be able to with a pen or pencil.

I think this project is a good example of how art can be made accessible by approaching it through the things that are interesting to you. It also helped to broaden my definition of art and pushed me think more about my other creative endeavors, such as collaging and sewing, which I have engaged with more this past year.

II. Artists with Ties to Biology and Math

As part of my exploration of art from a STEM viewpoint, I researched people embracing this interdisciplinarity in their art. The variety of media, styles, and focuses amazed me, so I have compiled some examples of past and present artists who use their work to understand, teach, elevate, and interact with math and biology.

Leonardo da Vinci (1452-1519)

Leonardo da Vinci did not have any formal education in biology or even in general, but through his independent studies he became a scientist, artist, and inventor along with many other pursuits.[1] One of his focuses was depicting anatomy. Da Vinci dissected animals and humans to study the muscular, skeletal, and organ structures of their bodies. As he continued this research, he became increasingly scientific in his objectives rather than following the more surface-level "artistic anatomy" movement of the time. He studied skulls and brains, aiming to identify sections responsible for different functions, such as common sense and rationality, and compared them with heads of other animals to try to find differences that could account for the behavior disparity between the subjects. Da Vinci also compared general body plans of humans to those of animals such as horses and birds, noting the similarities in leg, arm, and wing structures, and within the human species, he explored gestation and development from infancy to old age. As an artist, da Vinci reproduced his findings with incredible accuracy and detail, which was especially important in this time before photography and other imaging technology was available (Fig. 12).[2]

I knew very little about da Vinci before researching this - only that he was a renaissance artist. It was fascinating to read about how dedicated he was to anatomical research, especially because he didn't come from a scientific background. The precision in his anatomical drawings is amazing and knowing they originated from his own studies rather than from an interpretation of a doctor's research makes them all the more impressive.

Figure 12: "The skull sectioned and the seat of the *senso commune*"

Image from https://doi.org/10.1016/S0140-6736(19)30716-0

Anne Pratt (1806-1893)

Anne Pratt was a botanical artist and author in Victorian England. She developed an interest in botany through her mother, who was an avid gardener and through a family friend who was a botanist. Pratt wrote over twenty books with physical descriptions, illustrations, and general information on British plants using little technical vocabulary, making them an accessible resource for the general public to use in identifying local flora. The contents of these books were derived from her own observations and depictions of the plants she studied and from the teachings of formal botanists, and are largely written in a conversational tone with anecdotes from her life and others' relating to the species in question or the areas in which it is typically found. Her books became popular among a wide audience across the country, and Queen Victoria praised them, encouraging their distribution for education.[3]

Pratt produced a wide selection of drawings through her studies, including ferns, grasses, sedges, mosses, and flowers (Fig. 13). Part of what made her work so popular was that much of the botanical interest at the time was focused abroad, so people were excited to have their local plant life brought into the light with the same sophistication but in a manner everyone could connect with, regardless of their existing knowledge.[4]

Reading about Pratt was interesting because it shows one way women were able to become involved in a scientific field from within the confines of their societal roles; she was not a formal scientist, but an artist and writer, which were more acceptable past times for women. The fact that she became so successful and known for making beautiful and informative artwork makes her story even more compelling.

Figure 13: Pratt's drawing of a collection of British plants including varieties of thistle and knapweed

Image from: https://blog.biodiversitylibrary.org/2019/03/anne-pratt.html

David Goodsell

David Goodsell received a PhD in biochemistry from UCLA in 1987 and is now a computational biology professor at Scripps Research Institute and a molecular artist. Dr. Goodsell has written three books focusing on molecular biology and technology and writes a "Molecule of the Month" column for the RCSB Protein Data Bank, exploring the structure and function of a different molecule for each installment.[5] His current research lab focuses on modeling and analyzing cell structures and drug interactions with cells, particularly as it pertains to HIV resistance.[6]

Dr. Goodsell is also known for his art, where he compounds information from the RCSB Protein Data Bank, research papers, and molecular modeling programs to make accurate, informative, and colorful paintings of cells and their contents (organelles, proteins, enzymes, etc.). He starts his paintings with a sketch based on his research so the proportions and shapes are accurate, then paints it with watercolors (Fig. 14). Many of his pieces depict cellular processes, such as transcription and translation, while others show entire viruses.[7] The paintings often appear in biology education where they help students visualize and understand the interior and exterior of cells, while also being fascinating to look at purely from an artistic perspective, even if the viewer has limited biological knowledge.

Dr. Goodsell's artistic process is much more involved than mine was; making these paintings requires extensive research into molecular and cellular structure, and the paintings themselves are highly intricate and detailed to a level not seen in my graphs.

Figure 14: "HIV Translation and Frameshifting"

Images from https://journals.uic.edu/ojs/index.php/jbc/article/view/6627/5251

A collection of Dr. Goodsell's pieces can be found on the Protein Data Bank website at: http://pdb101.rcsb.org/sci-art/goodsell-gallery

Crochet Coral Reef Project

Margaret Wertheim is an artist and writer and Christine Wertheim is professor at CalArts and a published poet.[8] Together, they founded the Institute for Figuring, an organization focused on exploring interactions between art and science and making scientific and mathematical knowledge accessible to the public, mainly through museum installations and art exhibitions.[9] Through this organization, they started the Crochet Coral Reef Project, which uses crocheting techniques to replicate the texture and shape of coral in a durable, plush medium and create model coral reefs.[8]

Coral and many other forms of sea life are examples of natural hyperbolic geometry, which is mirrored in the construction of the models since they are derived from a hyperbolic crochet pattern that is elaborated and modified to create the frills and folds characteristic of coral (Fig. 15).[8]

Interestingly, it was long thought that hyperbolic geometry couldn't be represented in an interactive physical form, as any mathematically accurate models were too delicate to be useful. In 1997, however, Dr. Daina Taimina discovered that the shapes could be achieved with crocheting, and it remains one of the best physical representations of this type of geometry.[10]

Since the project's beginning, it has expanded globally as people participate in crocheting the reefs and raising awareness of the urgency with which they need to be protected. The project even gained attention from the Smithsonian, which launched the collaborative effort The Smithsonian Community Reef with participants from across the U.S. working to crochet a reef for display at the museum.[11]

Similar to my project, the Crochet Coral Reef Project uses mathematical concepts to create art that depicts biology. It's fascinating to read about how intertwined math and art can be and the diversity of ways it can happen.

Figure 15: Example of a crocheted coral reef.
Image from: https://www.margaretwertheim.com/crochet-coral-reef

Hunter Cole

Hunter Cole received her PhD in genetics from University of Wisconsin-Madison and is currently a biology professor at Loyola University Chicago, where she started the course *Biology through Art*, in which students create art in a biology lab, using lab equipment and resources. Dr. Cole's interests are focused on the interaction of art and biology, which she explores through a variety of media, one of which is bioluminescent bacteria. To do this, she paints the bacteria into petri dishes and assembles these dishes into a picture, which she photographs in the dark so the bacterial patterns are illuminated. She takes a series of photos of her paintings, showing the stages as the bacteria die over time until eventually there is only black (Fig. 16).[12, 13]

The paintings are largely biology-inspired, showing animals, cells, DNA, and the cycle of life. In other pictures, however, Dr. Cole attaches the petri dishes to frames wearable by human models, such as skirts and headdresses, then photographs them in the dark, either stationary or in motion, producing streaks of patterned light around the person. Other photos show people posing around the painted petri dishes, partially illuminated by the light of the bacteria.[12]

The bioluminescent paintings are a great example of how biology is a medium in which art can be made, and I love the idea of intertwining the media and the content - using biology to depict biology.

Figure 16: “Her Own DNA”
The left image is Dr. Cole’s painting in the light, the center is the first photo of it in the dark and the right is the 7th in the series of 8, after most of the bacteria has died.
Images from:
https://www.huntercole.org/artwork/living-light/living-drawings

Suzanne Anker

Suzanne Anker received her MFA from University of Colorado, Boulder and currently chairs the Fine Arts Department of the School of Visual Arts, New York.[14] Through her work at SVA, Anker created the Bio Art Lab, where students and faculty can use lab equipment to create art such as specimen displays and microscope photography.[15] She has also written widely about the interaction of biology and art in books and journals, and her work has been shown internationally in several museums.[14]

Anker makes a range of biology-based art including installments of plants grown in LED light, collections of natural and man-made items sorted by color in grids of petri dishes, porcelain and metallic sculptures of sea sponges, and pictures of butterflies overlaid on human brain MRIs.[14]

The most compelling to me is her collection “Remote Sensing”, which is a series of petri dishes containing small sculptures of plaster and resin made with a 3D printer. They are designed to evoke miniature landscapes such as forests and coral reefs (Fig. 17).[16]

Anker highlights the staggering degree of variety that exists within the field of biological art because she has created such a wide range of pieces in this realm. It is also notable that both Anker and Cole started programs at their institutions where students can create art in biology labs. It shows that this interdisciplinary movement is making opportunities to create STEM-based art increasingly accessible.

Figure 17: “Remote Sensing (35)”
Image from: http://suzanneanker.com/artwork/?wppa-album=21&wppa-photo=627&wppa-occur=1

Conclusion

These artists give a profile of the diversity that exists at the intersection of art and biology and the valuable outcomes of it, such as education, conservation, and human connection. These types of interactions extend far beyond biology; there are countless computer scientists, physicists, chemists, mathematicians, and other STEM professionals who create art based on their disciplines. Further, they extend beyond the individual, as collaborations between scientists and artists are increasingly common as a way for scientists to communicate and elevate their research and for artists to broaden the scope of their work and explore new perspectives. In these partnerships, both parties get to see into the others' world and learn about the skill and knowledge involved in each other's work .[17]

My graphs and these artists together emphasize the idea that art is a vast, complex field and everyone has the ability to create meaningful art by approaching it through their strengths and interests.

Acknowledgements

Thank you to my advisor, Dr. Amacher, for all her help, support, and enthusiasm throughout my project. I couldn't have done it without her!

Also, thank you to Teresa Brockmier, who helped me come up with the idea to graph animals and convinced me it could be a real project.

Lastly, thank you to all my friends and family who were so helpful in giving advice on my pictures and ideas for what animals to graph.

And Patty.. you know what you did

Works Cited

1. Sterpetti, A. V. (2019). Leonardo da Vinci (1452-1519): A Modern Scientist. *Circulation Research, 124*(5), 681-683. https://doi.org/10.1161/CIRCRESAHA.118.314391

2. Laurenza, D. (2019). Leonardo's contributions to human anatomy. *The Lancet, 393*, 1473-1476. https://doi.org/10.1016/S0140-6736(19)30716-0

3. Burns, M. (2017). Printing and publishing the illustrated botanical book in nineteenth century Great Britain. *Cogent Arts & Humanities, 4*(1). https://doi.org/10.1080/23311983.2017.1364058

4. Ferns of Great Britain: Anne Pratt. *The Garden Museum*. https://gardenmuseum.org.uk/ferns-of-great-britain-anne-pratt/

5. David S. Goodsell, PhD. *Scripps Research: Faculty*. https://www.scripps.edu/faculty/goodsell/

6. Goodsell, D. Molecular Art | Molecular Science. https://ccsb.scripps.edu/goodsell/

7. Goodsell, D. (2016). Cellular Landscapes in Watercolor. *Journal of Biocommunication*, *40*(1), 22-26. https://doi.org/10.5210/jbc.v40i1.6627

8. Wertheim, M. Science+Art Project: Crochet Coral Reef. https://www.margaretwertheim.com/crochet-coral-reef

9. The Institute for Figuring (website). https://theiff.org/about/about.html

10. Henderson, D. W. & Taimioa, D. (2001). Crocheting the Hyperbolic Plane. *Mathematical Intelligencer, 23*(2), 17-28. doi:10.1007/BF03026623

11. Marzec, C. (2010). When Art Meets Science: The Hyperbolic Crochet Coral Reef. *Smithsonian*. https://ocean.si.edu/ocean-life/invertebrates/when-art-meets-science-hyberbolic-crochet-coral-reef

12. Cole, H. Hunter Cole Website. https://www.huntercole.org

13. Cole, H. Hunter Cole: BioArtist. *Loyola University Chicago*. https://www.luc.edu/biology/aboutus/huntercolebioartist/

14. Anker, S. Artwork. http://suzanneanker.com/artwork/

15. SVA Bio Art Lab: Facilities. https://bioart.sva.edu/facilities/

16. Chambless, J. (2018). New ways of experiencing nature at the Brandywine. http://suzanneanker.com/blog/2018/07/09/new-ways-of-experiencing-nature-at-the-brandywine/

17. Gewin, V. (2021). How to Forge a Productive Science-Art Collaboration. *Nature, 590*, 515-518.

Appendices

Appendix A. Polar bear equations

1. $y = -0.015(x-2)^2 + 6.92 \ \{-12.99 < x < -10.16\}$
2. $y = (0.5x+5)^{0.5} + 4.97 \ \{x < -2.1\}$
3. $y = -0.087(x+5)^2 + 2.17 \ \{-5.15 < x < 3\}$
4. $y = \{2.52 < x < 3: 10x - 33.4\}$
5. $y = \left\{x < 5: -\left(0.5(x-2.52)^{0.5}\right) - 8.2\right\}$
6. $(x-5)^2 + 2(y+8.28)^2 = 1\{5 < x < 6\}\{y < -7.576\}$
7. $y = 15x - 84 \ \{5.095 < x < 5.4345\}$
8. $y = -1.8x + 7.3\{4 < x < 5.435\}$
9. $y = -6x\{0 < x < 0.7\}$
10. $y = 1.8(x+0.8)^2 - 8.2 \ \{-0.8 < x < 0.69\}$
11. $y = 0.8(x+3)^{0.5} - 9.39\{-0.66 > x > -2.9997\}$
12. $y = 0.08(x+1.5)^2 - 9.57\{-3 < x < 1.15\}$
13. $y = 0.45(x-0.1)^2 - 9.74\{0.1 < x < 2.65\}$
14. $y = 0.01(x-8)^2 - 3.3\{5.385 < x < 10\}$
15. $y = -7(x-9.6)\{9.7 < x < 10.6\}$
16. $y = 5^{(x-10.2)} - 9\{8.3 < x < 10.616\}$
17. $(x-8.3)^2 + 5(y+9.47)^2 = 1.3\{x < 8.4\}$
18. $y = 0.01(x-8.4)^2 - 9.98\{8.4 < x < 11.7\}$
19. $y = (x-11.6)^2 - 9.87\{11.7 < x < 13.3\}$
20. $y = -40(x-13.11)\{13.157 < x < 13.285\}$
21. $y = 3(x-13.157)^{0.5} - 2\{x < 13.2\}$
22. $y = -1.5x + 17.3\{13.18 < x < 16.6\}$
23. $1 = (x-15.8)^2 + (y+8.2)^2\{x > 16.59\}$
24. $1.66 = 0.5(x-16.66)^2 + 6(y+9.35)^2\{x < 16.59\}$
25. $y = 3^{(x-20)} - 9.9 \ \{16.6 < x < 19\}$
26. $20 = (x-14.82)^2 + (y+8)^2\{19.01 < x\}$
27. $y = -2x + 31.6\{18 < x < 19\}$
28. $30 = (x-22.92)^2 + (y+2)^2 \ \{x < 18\}$
29. $30 = (x-13.2)^2 + (y-3)^2\{x > 18\}\{y < 5.45\}$
30. $y = -0.055(x-11.45)^2 + 8\{7 < x < 17.865\}$
31. $y = 1.3(x+12)^2 + 2.28 \ \{-12.99 < x < -12\}$
32. $y = 0.06(x-5.5)^2 + 6.77\{4 < x < 7\}$
33. $y = 10(x+10.17)^2 + 4.7\{-10.17 < x < -10\}$
34. $y = -2(x+6.4)^2 + 6.23\{-6.8 < x < -5.8\}$
35. $y = -0.05(x-1.5)^2 + 7.23\{2.5 < x < 4\}$
36. $y = -0.05(x)^2 + 7.5\{0 < x < 2.5\}$
37. $y = -2(x+7.3)^2 + 6.5\{-7.78 < x < -6.93\}$
38. $2 = 5(x-18.2)^2 + (y-4)^2\{y < 4\}\{x < 18.67\}$
39. $y = -0.0461(x+5)^2 + 3 \ \{-9.15 < x < -5\}$
40. $y = 5(x+12.66)^2 + 3 \ \{-12.76 < x < -12.246\}$
41. $0.13 = (x+6)^2 + 3(y-4.95)^2$
42. $y = -0.005(x+5)^2 + 2.29\{-9.2 < x < -2.268\}$
43. $y = 0.02(x+7)^2 + 2.1\{-10 < x < -5\}$
44. $y = 0.3\sin(0.8(x+3.5)) + 2.46 \ \{-5 < x < -2.963\}$
45. $y = -0.05(x-0.87)^2 + 7.4 \ \{-2.1 < x < 4\}$
46. $y = 2.18 \ \{-5 < x < -2.134\}$
47. $y = 0.07(x+11)^2 + 2.21\{-12 < x < -10\}$
48. $y = \frac{1}{6(x+3.4)} - 9.788\{-2.997 < x < 0.1\}$
49. $(x-15.4)^2 + 1.2(y-3.81)^2 = 10\{x > 17.296\}\{y > 4.745\}$

Appendix B. Penguin equations

1. $16=6(x)^2+(y)^2\{-2.923<y<2.5\}$
2. $y=\frac{1}{0.2x}-1.4\{0.9935<x<1.28\}$
3. $y=-\frac{1}{0.5x}+0.94\{-0.88>x>-1.277\}$
4. $y=-1.3(x)^2+4.915\{0.1<x<.994\}$
5. $y=-0.3(x+0.39)^2+4.976\{-1.38<x<.12\}$
6. $y=0.3(x+1.6)^2+4.666\{-1.6<x<-1.38\}$
7. $y=0.05(x+2.2)^2+4.41\{-2.9<x<-1.799\}$
8. $y=-(x+1.792)^2+4.418\{-1.798<x<-.88\}$
9. $y=-0.015(x)^2+2.53\{-5.9<x<-1.28\}$
10. $y=-0.015(x)^2+2.53\{5.9>x>1.28\}$
11. $2.8=(x+4.9)^2+12(y-1.62)^2\{x<-5.1\}\{y<2.01\}$
12. $2.8=(x-4.9)^2+12(y-1.62)^2\{x>5.1\}\{y<2.01\}$
13. $y=-0.06(x+3)^2+1.26\{-4.23<x<-2.4\}$
14. $y=0.05(x+5)^2+1.14\{-5.1<x<-4.23\}$
15. $y=0.3(x+2.3)^2+1.238\ \{-2.4<x<-1.53\}$
16. $y=0.05(x-5)^2+1.14\{5.1>x>4.23\}$
17. $y=-0.06(x-3)^2+1.26\{4.23>x>2.4\}$
18. $y=0.3(x-2.3)^2+1.238\ \{2.4>x>1.53\}$
19. $y=0.3x^2-3\ \{-0.41<x<0.41\}$
20. $y=-0.01(x+1.83)^2+4.67\{-2.57<x<-1.5\}$
21. $y=-2(x+2.56)^2+4.665\{-2.9<x<-2.56\}$
22. $0.1=2(x+1.06)^2+(y-3.4)^2\{x>-.88\}$
23. $y=0.4\sin 35x-3.2\{y<-3.2\}\{-1<x<-0.5\}$
24. $y=-0.4\sin 35x-3.2\{y<-3.2\}\{1>x>0.5\}$
25. $y=9(x+0.93)^2-3.23\{-1.115<x<-.99\}$
26. $y=9(x-0.93)^2-3.23\{1.115>x>.99\}$
27. $y=0.6(x+0.85)^2-3.2\ \{-0.897<x<-0.809\}$
28. $y=0.6(x-0.85)^2-3.2\ \{0.897>x>0.809\}$
29. $y=0.6(x+0.67)^2-3.2\ \{-0.718<x<-0.629\}$
30. $y=0.6(x-0.67)^2-3.2\ \{0.718>x>0.629\}$
31. $y=3(x+0.8)^2-3.405\{-.538<x<-0.41\}$
32. $y=3(x-0.8)^2-3.405\{.538>x>0.41\}$
33. $y=-(x+0.75)^2-2.835\ \{-1.103<x<-.4106\}$
34. $y=-(x-0.75)^2-2.835\ \{1.103>x>.4106\}$

Appendix C. Salamander equations

1. $y = -(x)^2 + 8 \{-1 < x < 1\}$
2. $(x+1)^2 + (y-6.84)^2 = 0.03$
3. $(x-1)^2 + (y-6.84)^2 = 0.03$
4. $(x-0.83)^2 + 1.5(y-6.55)^2 = 0.1 \{6.45 < y < 6.69\} \{x > 0.75\}$
5. $(x-0.63)^2 + 3(y-6.5)^2 = 3 \{6.4 < y < 6.68\} \{x < 0\}$
6. $1.3(x-0.1)^2 + (y-5.9)^2 = 2.1 \{5 < y < 6.39\} \{x < 0\}$
7. $1.3(x+0.21)^2 + (y-6)^2 = 2.5 \{5 < y < 6.44\} \{x > 0\}$
8. $y = -(x+0.015)^2 + 5.77 \{0.87 < x < 1\}$
9. $y = -1.5(x+0.18)^2 + 5.77 \{-1.1 < x < -0.896\}$
10. $y = -0.06(x-1.7)^2 + 4.77 \{1 < x < 1.9\}$
11. $y = -0.07(x-4)^2 + 6.325 \{-2.063 < x < -1.099\}$
12. $(x-1.51)^2 + (y-5.26)^2 = 1 \{x > 2.499\} \{y < 5.45\}$
13. $0.6 = (x-1.967)^2 + 4(y-5.12)^2 \{2.49 > x > 2.2\} \{y > 5.096\}$
14. $(x-2.2)^2 + 3(y-5.57)^2 = 0.02 \{x < 2.2\}$
15. $y = -0.1(x-2.2)^2 + 5.653 \{2.2 < x < 2.45\}$
16. $y = -(x-1.526)^2 + 6.5 \{2.09 < x < 2.45\}$
17. $(x-2.175)^2 + (y-6.236)^2 = 0.01 \{y > 6.17\} \{x < 2.25\}$
18. $y = 2(x-2.8)^2 + 5.695 \{2.25 < x < 2.4\}$
19. $(x-2.833)^2 + (y-6.45)^2 = 0.007 \{y > 6.44\} \{x > 2.7505\}$
20. $y = (x-2.69)^2 + 5.93 \{2.4 < x < 2.64\}$
21. $y = -10(x-3.)^2 + 7$
22. $(x-0.313)^2 + (y-6.7)^2 = 6 \{x > 2.636\} \{y < 6.47\}$
23. $y = -3(x-3.637)^2 + 8 \{2.84 < x < 2.916\}$
24. $y = 13(x-3)^2 + 5.76 \{2.84 < x < 3\}$
25. $y = -0.06(x-6)^2 + 6.3 \{3 < x < 3.3\}$
26. $(x-3.33)^2 + (y-5.778)^2 = 0.008 \{x > 3.3\} \{y > 5.7\}$
27. $(x-4.64)^2 + (y-6.3)^2 = 3 \{y < 6.44\} \{x < 2.994\}$
28. $(x-4)^2 + (y-7.696)^2 = 5 \{y < 6.3\} \{x < 2.64\}$
29. $y = -4(x-3.39)^2 + 5.7 \{3.16 < x < 3.37\}$
30. $9(x-2.6)^2 + (y-5.9)^2 = 3 \{y < 5.49\} \{x > 2.58\}$
31. $y = -0.025(x-2.8)^2 + 4.17 \{1.3 < x < 2.6\}$
32. $(x-1.3)^2 + (y-3.973)^2 = 0.02 \{x < 1.3\} \{y > 3.95\}$
33. $5(x+1)^2 + (y+0.1)^2 = 40 \{y > -2.2\} \{x > 1.159\}$
34. $y = 20^{(2.1x-5.6)} + 4.76 \{1.886 < x < 2.5\}$
35. $y = x - 3.86 \{1.669 < x < 2.7\}$
36. $3(x-2.4)^2 + (y+1.)^2 = 0.3 \{x > 2.667\} \{y > -1.16\}$
37. $(x-2.76)^2 + (y+0.668)^2 = 0.01 \{y > -0.71\} \{x < 2.85\}$
38. $y = 50(x-2.88)^2 - 0.67 \{2.849 < x < 2.893\}$
39. $y = -5(x-2.761) \{2.77 < x < 2.893\}$
40. $(x-2.87)^2 + (y+0.035)^2 = 0.01 \{y > -0.05\} \{x < 2.965\}$
41. $3(x-4.235)^2 + (y-0.4)^2 = 5 \{y < 0\} \{x < 3.06\}$
42. $2(x-3.128)^2 + (y+0.5)^2 = 0.01 \{y < -0.5237\}$
43. $y = -(x-4.7854)^2 + 2 \{3.1966 < x < 3.5\}$
44. $(x-3.596)^2 + (y-0.32)^2 = 0.01 \{y > 0.347\}$
45. $y = 2.65(x-3.583) \{3.543 < x < 3.69\}$
46. $2(x-3.5)^2 + (y+0.53)^2 = 0.022 \{y < -0.653\} \{x < 3.54\}$
47. $y = -0.7(x-4.438)^2 - 0.1 \{3.53 < x < 4.05\}$
48. $(x-4.27)^2 + (y+0.1388)^2 = 0.01 \{x > 4.2\} \{y > -0.168\}$
49. $y = -0.1(x-9)^2 + 1.98 \{3.7 < x < 4.366\}$
50. $y = 0.91(x-4.274) \{3.54 < x < 4.2\}$
51. $y = 0.1(x-4) - 0.8 \{3.7 < x < 4.1\}$
52. $(x-4.1)^2 + (y+0.89)^2 = 0.01 \{x > 4.1\}$
53. $y = -0.9(x-4.1)^2 - 0.99 \{3.5 < x < 4.1\}$
54. $y = 0.6(x-1.98)^2 - 2.7 \{2.4 < x < 3.5\}$
55. $y = -0.6(x-3)^2 - 2.38 \{1.56 < x < 2.409\}$
56. $y = 0.3(x+3.4)^2 - 11 \{-3.4 < x < 1.556\}$
57. $(x+3.4)^2 + 1.2(y+9.42)^2 = 3 \{y < -8\} \{x < -3.4\}$
58. $(x+4.085)^2 + 2(y+8.15)^2 = 0.05 \{x > -4.19\} \{y > -8.2\}$
59. $(x+3.53)^2 + 1.2(y+9.4)^2 = 1.5 \{y < -8.55\} \{x < -4.3\}$
60. $y = -0.39(x+3.14)^2 - 8 \{-4.338 < x < -3.876\}$

Appendix C (cont.)

61 $y=0.6(x+3.5)^2-10.52\{-3.5<x<-0.77\}$

62 $y=0.39(x+3.5)^2-10.52\{-4.3<x<-3.5\}$

63 $y=(x-1.52)^2-10\{-0.96<x<-0.7277\}$

64 $(x+2.91)^2+(y+5.39)^2=5\ \{x>-0.78\ \}\{y<-4.9\}$

65 $y=-(x+1.886)^2-3\ \{-1.7<x<-0.9575\}$

66 $y=-0.3(x)-3.545\ \{-2.94<x<-1.695\}$

67 $5(x-1.95)^2+(y-1.5)^2=50\ \ \{y>1.1\}\{x<-1.052\}$

68 $6(x+2.28)^2+(y-0.77)^2=7\ \{y<1.045\}\{x>-1.366\}$

69 $1.5(x-0.24)^2+(y+1)^2=4\ \{x<-0.9\}\{y<-0.64\}$

70 $y=0.5(x+2.5)^2-2.76\{-3.593<x<-2.94\}$

71 $y=-0.5x-3.96\ \{-4.1<x<-3.595\}$

72 $(x+4.05)^2+(y+1.824)^2=0.01\{y>-1.918\}\{x<-4.016\}$

73 $y=-0.4x-3.337\ \{-4.016<x<-3.57\}$

74 $4(x+4.66)^2+(y+2.4)^2=5\ \{x>-3.8\}\{y>-1.907\}$

75 $(x+3.701)^2+(y+0.95)^2=0.01\ \{y>-0.975\}\{x<-3.615\}$

76 $y=-(x+5.316)^2+2\ \{-3.612<x<-3.43\}$

77 $2.6(x+3.281)^2+(y+1.5)^2=0.06\ \{y<-1.5587\}\{x<-3.2\}$

78 $y=5(x+3.3)^2-1.7572\ \{-3.206<x<-2.857\}$

79 $(x+2.76)^2+(y+0.8)^2=0.01\ \{y>-0.782\}$

80 $4(x+1.663)^2+(y+2.2)^2=6\ \{y>-1.594\}\{x<-2.662\}$

81 $2.8(x+2.7653)^2+(y+1.5822)^2=0.02\ \{y<-1.5936\}\{x<-2.7483\}$

82 $y=0.8(x+3.222)^2-1.9\ \{-2.7489<x<-2.3\}$

83 $(x+2.213)^2+(y+1.27)^2=0.01\{x>-2.3\}\{y>-1.3\}$

84 $y=-(x+1.57)^2-1\ \{-2.53<x<-2.118\}$

85 $y=0.4x-0.909\ \{-2.5296<x<-2.25\}$

86 $(x+2.22)^2+(y+1.9045)^2=0.01\{x>-2.2514\}\{y>-1.96\}$

87 $y=0.5x-0.891\ \{-2.54<x<-2.136\}$

88 $y=-0.163x-2.58\ \{-2.54<x<-1.01\}$

89 $8(x-0.325)^2+(y)^2=20\ \{y<1.1\}\{x<-1.008\}$

90 $(x-3.596)^2+(y-0.32)^2=0.01\{y<0.347\}\{x>3.69\}$

91 $(x-5.17)^2+(y+0.7)^2=3\ \{x<3.543\}\{y>-0.6503\}$

92 $y=6(x+2.07)^2+3.75\ \{-2.3<x<-2.07\}$

93 $y=0.1(x+1.)^2-2.4\ \{-2.54<x<-1.009\}$

94 $(x+1.986)^2+2(y-4.1)^2=0.1\ \{y>4.058\}\{x<-2.17\}$

95 $y=-0.1(x-0.9)^2+5.225\ \{-2.177<x<-1.97\}$

96 $(x+2.03)^2+(y-4.482)^2=0.01\{y>4.398\}\{x>-2.08\}$

97 $y=-0.4(x+1.5)^2+4.704\ \{-2.3<x<-2.08\}$

98 $(x+0.301)^2+(y-4.5)^2=4\ \{y>4.446\}\{x<-2.2\}$

99 $(x+2.2975)^2+(y-5.15)^2=0.01\ \{y>5.127\}\{x>-2.38\}$

100 $(x+4.58)^2+(y-5.6)^2=5\ \{y<5.209\}\{x>-2.557\}$

101 $(x-0.003)^2+(y-6.5)^2=10\ \{y<5.246\}\{x<-2.558\ \}$

102 $(x+2.975)^2+(y-5.18)^2=0.01\ \{x<-2.9\}\{y>5.13\}$

103 $(x+5.501)^2+(y-3.7)^2=8\ \{x>-3.06\}\ \{y>4.5\}$

104 $(x+3.2)^2+(y-1.702)^2=8\ \{y>4.5\}\{x>-3.13\}$

105 $(x+3.13)^2+(y-4.43)^2=0.01\ \{x<-3.13\}$

106 $y=-3(x+3.13)^2+4.33\ \{-3.13<x<-2.6\}$

107 $y=3(x+2.26)^2+3.15\ \{-2.609<x<-2.26\}$

108 $y=0.43(x+2.2)^2+3.15\{-2.25<x<-1.05\}$

Appendix D. Trout equations

1. $y=-0.02(x)^2+2\ \{-6<x<-0.3\}$
2. $y=-0.016(x)^2+2\ \{2.64<x<6.8\}$
3. $y=-0.95(x-1.25)^2+4.3\ \{-0.303<x<0.655\}$
4. $y=-0.56(x-1)^2+4.334\ \{1.816<x<2.04\}$
5. $(x-3.28)^2+2(y-2.487)^2=0.07\ \{x>3.28\}\{y<2.58\}$
6. $y=-(x-3.28)^2+2.3\ \{2.65<x<3.28\}$
7. $y=0.4(x-6.8)^2+1.26\{6.8<x<7.4\}$
8. $y=0.49x-2.22\ \{7.395<x<10.5\}$
9. $y=-0.182(x-1)^2+3.718\ \{2.99<x<3.525\}$
10. $y=0.26(x-4)^2+2.73\ \{2.04<x<2.7\}$
11. $y=-(x-10.2)^2+3.056\{9.989<x<10.2\}$
12. $(x-10.2)^2+(y-2.956)^2=0.01\ \{x>10.2\}\{y>2.99\}$
13. $y=0.9(x-9)^2-0.36\ \{9.9<x<10.82\}$
14. $(x-10.108)^2+(y-0.24)^2=0.06\ \{x<9.9\}$
15. $y=-1.1(x-9.6)^2+0.21\ \{9.9<x<10.9\}$
16. $(x-10.57)^2+1.2(y-2.694)^2=0.068\ \{10.446<x\}\{y>2.62\}$
17. $(x-10.645)^2+1.2(y+1.7)^2=0.068\ \{x>10.56\}\{y<-1.65\}$
18. $y=-0.3x+1.243\{7.395<x<10.56\}$
19. $y=0.2(x-7.9)^2-1.026\ \{7<x<7.41\}$
20. $(x-6.96)^2+(y+1)^2=0.02\ \{x<7.012\}\{y>-0.94\}$
21. $y=-2.7(x-7.301)^2-0.35\ \{6.326<x<6.84\}$
22. $1.3(x-6.089)^2+(y+2.896)^2=0.07\ \{y<-2.96\}\{x>5.97\}$
23. $y=0.02(x-1)^2-2\ \{1.5<x<3.92\}$
24. $y=0.144(x-7.14)^2-3.32\ \{3.92<x<5.97\}$
25. $(x-6.3)^2+1.5(y-1.46)^2=0.08\ \{y>1.32\}\{x>6.3\}$
26. $y=-0.2(x-6.2)^2+1.6924\ \{5.34<x<6.3\}$
27. $y=0.25(x-2.52)^2-2.25\{1.5<x<2.4\}$
28. $y=0.011(x+4)^2-2.2\ \{-4<x<0.24\}$
29. $(x-2.35)^2+3(y+2.425)^2=0.1\ \{x>2.39\}\{y>-2.5\}$
30. $y=1.27(x-1.952)^2-3.1\ \{1.95<x<2.64\}$
31. $y=1(x-1.29)^2-3.092\ \{0.24<x<1.1\}$
32. $y=0.18(x-1.67)^2-3.115\ \{1.098<x<1.96\}$
33. $y=-0.025(x+3)^2+1.5\ \{-8<x<-6\}$
34. $y=0.8(x+9.2)^{0.5}-0.15\ \{x<-6\}$
35. $1.2(x+8.4)^2+(y+0.094)^2=0.5\ \{x<-8.647\}\{y>0.17\}$
36. $y=-1.1(x+8.977)^2-0.286\{-8.81<x<-8.3\}$
37. $y=-0.8(x+8.8)^2-0.59\ \{-8.95<x<-8.3\}$
38. $y=0.3(x+9)^2-0.636\ \{-9.07<x<-8.69\}$
39. $y=-2(x+9.05)^2-0.634\ \{-9.2<x<-9.07\}$
40. $y=3(x+8.56)^2-1.056\ \{-8.95<x<-8.62\}$
41. $y=0.019(x+1)^2-2.37\ \{-9<x<-4\}$
42. $1(x+3.5)^2+(y-9.62)^2=140\ \{x<-4\}\{y<-1.05\}$
43. $(x+1.8)^2+(y+13.65)^2=247\ \{y>0.556\}\ \{x<-0.303\}$
44. $(x+5)^2+2(y+1.1)^2=20$
45. $2(x+8.81)^2+(y)^2=0.1\ \{y<-0.153\}\{x<-8.81\}$
46. $(x+4.8)^2+1.8(y+1.3)^2=0.5\{x>-4.2\}$
47. $(x-1.04)^2+1.4(y-0.9)^2=10\ \{x>2.668\}\{y>2.547\}$
48. $y=-0.4(x+7.8)^2+0.762\ \{-8.7<x<-8.5\}$
49. $(x+8.3)^2+(y+0.145)^2=0.5\ \{y>-0.15\}\{x<-8.69\}$
50. $(x-1.234)^2+1(y-3.212)^2=0.9\ \{y>3.96\}$
51. $(x-9.12)^2+(y+2.6)^2=8\ \{y>-2.96\}\{x<6.86\}$
52. $y=0.23(x+3.47)^2-1.7\{-4.198<x<-3\}$
53. $(x+1)^2+(y+7.33)^2=50\ \{y>-1.02\}\{x<-1.9\}$
54. $(x+1.9)^2+1.3(y+0.493)^2=0.04\ \{x>-1.9\}\{y>-0.55\}$
55. $(x+3.7)^2+(y-0.475)^2=5\ \{y<-0.539\}\{x>-3\}$
56. $(x+7)^2+2(y+0.5)^2=5\ \{x>-6.05\}\{y<0.3\}$
57. $(x+8.8)^2+1.3(y)^2=8\ \{x>-6.79\}\{y<0\}$

Appendix E. Turtle equations

1. $3(x)^2+(y)^2=5\ \{-0.513<x<0.513\}\{y>0\}$
2. $1.3(x)^2+(y)^2=5.264\ \{-2.052<y<-0.547\}$
3. $(x-0.85)^2+1.2(y+0.21)^2=8\ \{y>-0.547\}\{x<-0.511\}$
4. $(x+0.85)^2+1.2(y+0.21)^2=8\ \{y>-0.547\}\{x>0.511\}$
5. $y=0.6\,\mathrm{abs}(x)-2.59\ \{-2.54<y<-2.053\}$
6. $(x)^2+(y+2.4251)^2=0.02\ \{y<-2.545\}$
7. $(x-2.1)^2+4(y-1.83)^2=1.64\ \{y>1.872\}\{x<2.01\}$
8. $(x-2.03)^2+3.7(y-1.68)^2=2.3\ \{x>2.01\}\{y>1.945\}$
9. $(x-3.375)^2+1.5(y-1.9)^2=0.01\{x>3.375\}\{y<1.945\}$
10. $y=0.03(x-3.3)^2+1.817\ \{2.4<x<3.37\}$
11. $(x-2)^2+5.6(y-1.526)^2=0.66\ \{y>1.79\}\{x<2\}$
12. $2(x-1.5)^2+(y-1.475)^2=0.1\ \{y>1.4846\}\{x<1.4836\}$
13. $y=-0.8(x+1.7)^2+2.1\ \{-1.7<x<-1.018\}$
14. $(x+1.7)^2+(y-0.367)^2=3\ \{x<-1.7\}\{y>1\}$
15. $1.5(x+3.255)^2+(y-0.963)^2=0.006\ \{y<1\}\{x<-3.218\}$
16. $y=-0.2(x+3)^2+1.76\ \{-1.67<x<-1.457\}$
17. $1.2(x+1.1)^2+(y+2.923)^2=20\ \{y>0.9\}\ \{x<-1.98\}$
18. $x=-0.35\ \{-3.03<y<-2.38\}$
19. $y=-20x+6.5\ \{0.441<x<0.4816\}$
20. $y=-1.5(x-0.4)^2-3.192\ \{0.549<x<0.887\}$
21. $2(x-1.227)^2+(y+3.4899)^2=0.015\ \{y<-3.495\}\{x>1.22\}$
22. $(x+3.13)^2+(y+4)^2=20\ \{y>-3.493\}\ \{x>0.896\}$
23. $y=-0.6(x-1.0)^2-3.583\ \{0.9605<x<1.218\}$
24. $(x-0.964)^2+(y+3.484)^2=0.01\ \{y<-3.548\}\{x<0.964\}$
25. $(x-0.62)^2+(y+3.103)^2=0.02\ \{y<-3.13\}\{x<0.549\}$
26. $(x+1.872)^2+2(y-1.234)^2=0.1\ \{x>-1.9868\}\{y>1.4\}$
27. $y=-0.9(x)^2-2.99\ \{-0.77<x<-0.4\}$
28. $y=-1.5(x+0.7)^2-3.55\ \{-0.992<x<-0.819\}$
29. $(x-3.352)^2+1.4(y+3.5)^2=20\ \{y>-3.584\}\{x<-0.8\}$
30. $2(x+1.033)^2+(y+3.57)^2=0.015\{y<-3.584\}\{x<-0.992\}$
31. $(x+0.491)^2+(y+3.0254)^2=0.02\ \{x>-0.41\}\{y<-3.01\}$
32. $(x+0.86)^2+(y+3.48)^2=0.01\ \{x>-0.83\}\{y<-3.524\}$
33. $y=-2(x-0.1)^2+3.607\ \{-0.3<x<0.424\}$
34. $2(x+0.8)^2+(y-2.48)^2=0.4\ \ \{x>-0.442\}\{y<2.808\}$
35. $1.3(x-0.2)^2+(y-2.87)^2=0.5\ \ \{x<-0.284\}\{y>2.8\}$
36. $(x+0.0683)^2+(y-3)^2=0.4\ \{x>0.399\}\{y>2.8\}$
37. $3(x-0.795)^2+(y-2.37)^2=0.4\ \{y>2.08\}\{x<0.5376\}$
38. $1.3(x-2.2)^2+(y+0.383)^2=5\ \{x<2.4\}\{y>1.552\}$
39. $(x-0.8)^2+1.1(y)^2=6\ \{y>-2\}\{x<-0.15\}$
40. $(x+0.8)^2+1.1(y)^2=6\{y>-2\}\{x>0.15\}$
41. $y\ =-\mathrm{abs}(2x)-1.05\ \{-2.24<y<-2\}$
42. $y=-2\ \{-0.465<x<0.465\}$
43. $y=-\mathrm{abs}(1.5x)-0.2\ \{-1.897<y<-1.631\}$
44. $y=-\mathrm{abs}(x)+0.1\ \{-1.456<y<-1.2\}$
45. $y=-\mathrm{abs}(0.8x)+0.5\ \{-0.96<y<-0.723\}$
46. $y=-\mathrm{abs}(0.5x)+0.65\ \{-0.337<y<-0.171\}$
47. $y=-\mathrm{abs}(0.3x)+0.9\ \{0.325<y<0.417\}$
48. $y=-\mathrm{abs}(0.1x)+1.05\ \{0.879<y<0.904\}$
49. $y=1.3\ \{-1.444<x<-1.235\}$
50. $y=1.3\ \{1.444>x>1.235\}$
51. $y=\mathrm{abs}(0.3x)+1.3\ \{1.596<y<1.638\}$
52. $y=\mathrm{abs}(0.5x)+1.5\ \{1.849<y<1.894\}$
53. $y=-(x)^2+2.175\ \{-0.1519<x<0.1519\}$
54. $y=-(x)^2-1.1\ \{-0.812<x<0.812\}\{y<-1.16\}$
55. $y=-0.3(x)^2-0.5\ \{-1.069<y<-0.65\}$
56. $y=-0.2(x)^2+0.5\ \{-0.044<y<0.4\}$
57. $y=0.3(x)^2+1.75\ \{1.879>y>1.78\}$
58. $y=-\mathrm{abs}(0.2x)+1.3\ \{1.019<y<1.159\}$
59. $y=1.1\,\mathrm{abs}(x)-1.43\ \{-0.6507>y>-1.16\}$
60. $y=-1.5\,\mathrm{abs}(x)+0.41\ \{-0.652<y<-0.14\}$
61. $y=1.59\,\mathrm{abs}(x)-0.73\ \{-0.143<y<0.399\}$
62. $y=-1.3\,\mathrm{abs}(x)+1.32\ \{0.4<y<0.8\}$
63. $y=1.2\,\mathrm{abs}(x)+0.32\ \{0.8<y<1.159\}$
64. $y=-1.6\,\mathrm{abs}(x)+2.286\ \{1.1586<y<1.78\}$
65. $y=1.78\ \{-0.316<x<0.316\}$
66. $y=0.8\ \{-0.395<x<0.385\}$
67. $y=-0.14\{-0.367<x<0.367\}$
68. $y=-1.16\ \{-0.245<x<0.245\}$

Appendix F. Slug equations

1. $y=0\ \{-6<x<5.5\}$
2. $y=-0.04(x+0.07)^2+1.562\{-6.088<x<-0.943\}$
3. $(x+6)^2+5(y-0.064)^2=0.02\ \{x<-6\}\{y<0.115\}$
4. $(x)^2+3(y+4.5709)^2=100\ \ \{0.3>x>-0.838\}\{y>1.18\}$
5. $y=0.5(x-5.5)^2\ \{5.5<x<6\}$
6. $2(x-6.05)^2+(y-0.025)^2=0.015\ \{y>0.125\}\{x<6.08\}$
7. $y=-0.4x+2.572\ \{6.068<x<6.25\}$
8. $(x-6.28)^2+(y-0.1677)^2=0.01\{x>6.244\}\{y<0.25\}$
9. $y=-0.5x+3.418\ \{6.23<x<6.339\}$
10. $(x-6.57)^2+2(y-0.606)^2=0.3\ \{x<6.23\}\{y<0.69\}$
11. $y=0.6x-2.93\ \{6.036<x<7.07\}$
12. $y=0.55x-2.44\ \{5.812<x<7\}$
13. $(x-7.12)^2+(y-1.42)^2=0.015\ \{y>1.409\}$
14. $(x-7.12)^2+(y-1.42)^2=0.015\ \{x>7.07\}\{y<1.409\}$
15. $y=1.3x-6.6\ \{5.75<x<6.49\}$
16. $y\ =\ 1.2x-5.75\{5.55<x<6.387\}$
17. $(x-6.5)^2+(y-1.96)^2=0.015\ \{y>1.914\}$
18. $(x-6.5)^2+(y-1.96)^2=0.015\ \{x>6.5\}\{y<1.914\}$
19. $y=-0.0208(x)^2+1.55\ \{-0.943<x<5.54\}$
20. $(x-5.8)^2+(y-0.826)^2=0.005\ \{y<0.8787\}\{x<5.81\}$
21. $(x-2.8)^2+1.8(y-1.95)^2=5.5\ \{y<1.544\}\{x<4.823\}$
22. $(x-6.3)^2+(y-0.45)^2=0.006\ \{x>6.28\}$
23. $y=0.3x-1.36\ \{6.063<x<6.289\}$
24. $y=0.3x-1.52\ \{6.18<x<6.33\}$

Appendix G. Meerkat equations

1. $y = -0.15(x-1)^2 + 7.5 \ \{0 < x < 1.735\}$
2. $(x-1.4)^2 + (y-6.045)^2 = 2 \ \{x > 1.735\}\{y > 6.7\}$
3. $(x+2)^2 + (y-13.35)^2 = 40 \ \{x > -1.28\}\{y < 7.35\}$
4. $(x+1.25)^2 + (y-6.845)^2 = 0.05 \ \{x < -1.28\}\{y > 6.87\}$
5. $y = 4(x+1.78)^2 + 6.5 \ \{-1.7 < x < -1.47\}$
6. $(x+5.32)^2 + (y-12.6)^2 = 50 \ \{y < 6.528\}\{x > -3.2\}$
7. $(x+3)^2 + 2(y-5.495)^2 = 0.3 \ \{x < -3.177\}\{y > 5.32\}$
8. $(x+1.8)^2 + (y-6.391)^2 = 4 \ \{y < 5.327\}\{x < -3.225\}$
9. $y = 3(x+2.95)^2 + 4.761 \ \{-3.225 < x < -2.95\}$
10. $(x+1.68)^2 + 4(y-6.209)^2 = 10 \ \{y < 4.761\}\{x < -1\}$
11. $3(x-2.6)^2 + (y-6.16)^2 = 0.3 \ \{x > 2.653\}\{y > 5.74\}$
12. $(x-3.05)^2 + (y-5.565)^2 = 0.05 \ \{x < 2.872\}\{y > 5.53\}$
13. $3(x+1.156)^2 + (y-4)^2 = 50 \ \{3.2 < y < 5.74\}\{x > 0\}$
14. $y = -4(x+1)^2 + 4.665 \ \{-1 < x < -0.45\}$
15. $1.8(x+1.93)^2 + (y-0.4)^2 = 50\{x > 2.9\}\{y > -1\}$
16. $5(x-7.385)^2 + (y+3)^2 = 90 \ \{y < -0.954\}\{x < 4.5\}$
17. $2(x+0.2)^2 + (y-2.67)^2 = 0.7 \ \{x < -0.45\}\{y > 2.6\}$
18. $3(x+1.19)^2 + (y-2.5)^2 = 0.5 \ \{y < 2.6\}\{x > -0.95\}$
19. $1.2(x-4.696)^2 + (y+1.5)^2 = 50\{x < -0.949\}\{y > -2\}$
20. $2(x-3.24)^2 + (y+12.4)^2 = 1.8 \ \{y > -12.637\}\{x < 3\}$
21. $(x-15)^2 + (y+3.64)^2 = 150 \ \{y < -9.9\}\{x < 11\}$
22. $2(x-10.2)^2 + (y+3.02)^2 = 150 \ \{x < 10.998\}\{y < -11\}$
23. $2(x+2.6)^2 + (y+14)^2 = 50 \ \{x > 2.278\}\{y < -12.5\}$
24. $3(x-3.8)^2 + (y+17.3)^2 = 10 \ \{y > -17.5\}\{x < 2.278\}$
25. $y = 8.1x - 34 \ \{1.963 < x < 2.363\}$
26. $(x-1.68)^2 + (y+18)^2 = 0.09 \ \{x > 1.68\}\{y < -18\}$
27. $y = 0.2x - 18.64 \ \{0.6 < x < 1.7\}$
28. $(x-0.56)^2 + 2(y+18.363)^2 = 0.05 \ \{x < 0.6\}\{y < -18.3\}$
29. $4(x-0.1)^2 + (y+16.37)^2 = 4 \ \{x > 0.352\}\{y < -15.3\}$
30. $(x-8.4)^2 + 1.1(y+10.6)^2 = 80 \ \{x < 0.952\}\{y < -9\}$
31. $(x-4.24)^2 + 1(y+4.83)^2 = 39 \ \{x < -0.39\}\{y < -3.909\}$
32. $y = 1.4x + 0.44 \ \{-2.5 < x < -0.4\}$
33. $3(x+2.285)^2 + (y+3.57)^2 = 0.4 \ \{x < -2.493\}\{y > -3.63\}$
34. $2(x-2.34)^2 + (y+4)^2 = 50 \ \{y < -3.554\}\{x < -2.55\}$
35. $3(x+2.295)^2 + (y+5.39)^2 = 0.2 \ \{y < -5.47\}\{x < -2.1\}$
36. $1.7(x+3.74)^2 + (y+5)^2 = 5 \ \{x > -2.113\}\{y < -5\}$
37. $5(x+0.617)^2 + (y+5)^2 = 10 \ \{y > -5.16\}\{x < -1.95\}$
38. $(x+0.98)^2 + (y+4.2)^2 = 1 \ \{y > -3.98\}\{x < -1.6\}$
39. $(x+6)^2 + (y-3.7)^2 = 70 \ \{x > -1.604\}\{y < 0.3\}$
40. $2(x+0.6)^2 + (y-0.22)^2 = 0.2 \ \{x > -0.39\}\{y < 0.3\}$
41. $6(x+2.33)^2 + (y+17)^2 = 50 \ \{x > 0.336\}\{y > -16.359\}$
42. $1.3(x-0.21)^2 + (y+18.2)^2 = 0.06 \ \{x < 0.341\}\{y < -18.148\}$
43. $y = 6(x)^2 - 18.14\{0 < x < 0.555\}$
44. $y = 0.073(x-15)^2 - 18 \ \{4.492 < x < 15\}$
45. $(x-15)^2 + (y+18.316)^2 = 0.1\{x > 15\}$
46. $y = 0.0605(x-15)^2 - 18.63 \ \{3.814 < x < 15\}$
47. $y = -0.6(x-3.45)^2 - 10.98 \ \{2.997 < x < 3.814\}$

Appendix H. Hippo equations

1. $y=-0.07(x+2.3)^2+5.3\ \{-7.271<x<-2\}$
2. $(x+7.19)^2+(y-2.8)^2=0.6\ \ \{x<-7.27\}\{y>3\}$
3. $(x+8.76)^2+(y-3.57)^2=1\ \{y<3\}\{x>-8.5\}$
4. $(x+8.5)^2+(y-2.34)^2=0.07\ \{x<-8.5\}\{y>2.25\}$
5. $2(x+9)^2+(y-1.98)^2=0.2\ \{x>-8.75\}$
6. $2(x+7.3)^2+(y+1.25)^2=13\ \{x<-8.74\}\{y>0.247\}$
7. $1.2(x+8.74)^2+(y+0.44)^2=1.4\ \{y<0.247\}\{x<-8.299\}$
8. $(x+8.3)^2+(y+0.52)^2=1\{x>-8.3\}\{y<-1.06\}$
9. $(x+8.9)^2+(y+0.1)^2=3\ \{y<0\}\{x>-7.458\}$
10. $(x+6.95)^2+(y)^2=0.05\ \{x<-6.83\}\{y>0\}$
11. $(x+6.55)^2+1.3(y-0.6)^2=0.3\{x>-6.82\}\{y<0.537\}$
12. $1.4(x+5.76)^2+(y-1)^2=0.3\ \{y<0.537\}\{x<-5.5\}$
13. $3(x+6.59)^2+(y+2)^2=10\{x>-5.5\}\{y>-1.67\}$
14. $(x+5.32)^2+2(y+1.644)^2=0.3\ \{x>-5.205\}\{y<-1.674\}$
15. $(x+5.15)^2+(y+2.215)^2=0.04\ \{x<-5.198\}$
16. $(x+4.7)^2+10(y+1.99)^2=2\ \{-5.2<x<-4.038\}\{y<-2\}$
17. $(x+4.1)^2+(y+2.195)^2=0.04\ \{x>-4.1\}\{y<-2.25\}$
18. $y=15x+56.4\ \{-3.911<x<-3.74\}$
19. $(x+3.517)^2+(y-0.29)^2=0.05\ \ \{y>0.29\}\{x<-3.4\}$
20. $(x-0.8)^2+(y-2)^2=20\ \{y<0.485\}\{x<-2\}$
21. $(x+2.3)^2+(y+1.819)^2=0.2\ \{y<-1.48\}\{x>-2.1\}$
22. $(x+2.1)^2+2(y+2.34)^2=0.03\ \{x<-2.1\}$
23. $(x+1.72)^2+2(y+1.5)^2=2\ \{-2.104<x<-1.23\}\{y<-2\}$
24. $(x+1.31)^2+(y+2.23)^2=0.05\ \{x>-1.224\}\{y<-2.189\}$
25. $3(x+7.4)^2+(y-3)^2=0.13\ \{y>3\}$
26. $5(x+7.7)^2+(y-3.35)^2=0.1\ \{y>3.234\}\{x<-7.7\}$
27. $5(x+7.7)^2+(y-3.35)^2=0.1\ \{y>3.475\}$
28. $y=0.4(x-0.5)^2-2.2\ \{-1.85<x<-1.44\}$
29. $(x-0.2)^2+3(y-2.94)^2=30\ \{y<0\}$
30. $(x)^2+10(y-6.906)^2=30\ \{y<5.294\}\{x<0.5\}$
31. $(x-1)^2+2(y-2.4)^2=15.78\{x>0.423\}\{y>1.85\}$
32. $(x-10)^2+(y+1.99)^2=40\ \{x<4.75\}\{y>-0.896\}$
33. $(x-4.2)^2+(y-1.98)^2=0.5\ \{x>4.746\}\{y<1.845\}$
34. $(x-3.334)^2+(y+0.8)^2=0.2\ \{y<-0.896\}\{x>3.7\}$
35. $y=2x-8.46\ \{3.159<x<3.7\}$
36. $(x-2.25)^2+(y+0.31)^2=0.1\ \{x>2.24\}\{y>-0.322\}$
37. $x=2.566\ \{-0.32>y>-1\}$
38. $y=3.8(x-2)^2-2.2\{2<x<2.56\}$
39. $(x-2)^2+(y+2.34)^2=0.02\ \{x<2\}$
40. $y=-2.481\ \{1.989<x<2.649\}$
41. $(x-2.598)^2+(y+1.85)^2=0.4\{x>2.641\}\{y<-2.123\}$
42. $y=-9.4(x-3.83)^2-0.6\ \{3.83<x<4.21\}$
43. $(x-4.1)^2+2(y+2.25)^2=0.04\ \{x<4.1\}$
44. $(x-4.2)^2+(y-0.77)^2=10\{4.96>x>4.067\}\{y<0\}$
45. $5(x-2.6)^2+(y+0.9)^2=30\ \ \{x>4.968\}\{y<0.034\}$
46. $2(x-4.112)^2+(y+1.967)^2=0.02\ \{x>4.1\}\{y<-1.9548\}$
47. $y=-1.2x+6.19\ \{4.387<x<4.915\}$
48. $10(x-4.9)^2+(y-2)^2=0.7\ \{4.9>x>4.698\}\{y<2\}$
49. $10(x-4.9)^2+(y-2)^2=0.7\ \{x>4.925\}$
50. $2(x+2.67)^2+(y+2.1)^2=5\ \{x>-1.442\}\{y>-2.193\}$
51. $(x-4.7)^2+(y-0.061)^2=0.1\ \{x>4.912\}\{y>0.0257\}$

Appendix I. House cat equations

1 $(x+3)^2+1.4(y-2)^2=3\ \{x>-4.2\}\{y>2.9\}$

2 $(x+1.3027)^2+(y-3.2)^2=0.2\ \{x<-1.06\}\{y<2.9\}$

3 $(x-4.79)^2+5(y-1.048)^2=50\{y>2.824\}\ \{x<6.2\}$

4 $y=0.1(x-6.6)^2+4.13\ \{6.175<x<10.5\}$

5 $(x-11.35)^2+(y-5.753)^2=0.113\ \{11.151<x\}\{y>5.5\}$

6 $y=0.11(x-6.6)^2+3.133\ \{6.9<x<10.794\}$

7 $(x-6.8)^2+(y-2.898)^2=0.04\ \{x<6.819\}\{y>2.794\}$

8 $(x-11.27)^2+(y-4.482)^2=2\ \{10.267<x<11.299\}\{y>4\}$

9 $(x-12.7)^2+(y-2.07)^2=12\ \{10.709<x<11.39\}\{y>4\}$

10 $y=-0.5(x-6.4)^2+2.83\ \{6.614<x<8\}$

11 $-0.28\left(\frac{x-5.6}{3}\right)^2+0.05\left(\cos\left(x(8)\right)\right)+4.22\ \{-0.967<x<5.79\}$

12 $7\left(\frac{x+1.3}{3}\right)^2+0.02\left(\cos\left(x(20)\right)\right)+2.782\ \{-1.604<x<-0.954\}$

13 $-2.5\left(\frac{x+3.04}{3}\right)^2+0.06\left(\cos\left(x(8)\right)\right)+3.39\ \{-4.41<x<-1.6\}$

14 $0.6\left(\frac{x-6}{3}\right)^2+0.05\left(\cos\left(x(5)\right)\right)+4.22\ \{5.79<x<11.15\}$

15 $0.696\left(\frac{x-6}{3}\right)^2+0.05\left(\cos\left(x(5)\right)\right)+3.09\ \ \{6.807<x<11.6\}$

16 $-5\left(\frac{x-6.4}{3}\right)^2+0.05\left(\cos\left(x(8)\right)\right)+2.87\{6.629<x<8\}$

17 $05\left(\frac{x-12.56}{3}\right)^2+0.1\left(\cos\left(x(20)\right)\right)-10\{7.996<x<8.4\}$

18 $(x-8.12)^2+(y+0.5)^2=0.09\ \{y<-0.38\}\{x>7.95\}$

19 $7\left(\frac{x-9.21}{3}\right)^2+0.1\left(\cos\left(x(18)\right)\right)-2\ \{7.2<x<7.955\}$

20 $20\left(\frac{x-6}{3}\right)^2+0.1\left(\cos\left(x(25)\right)\right)-2.07\ \{6.9<x<7.2\}$

21 $(x-6.591)^2+(y+0.3)^2=0.1\{y<-0.363\}$

22 $-30\left(\frac{x-8.05}{3}\right)^2+0.05\left(\cos\left(x(50)\right)\right)+10\ \{6.282<x<6.4\}$

23 $(x-6.084)^2+3(y-0.97)^2=0.1\ \{y<0.95\}$

24 $0.15\left(\frac{x+2}{3}\right)^2+0.05\left(\cos\left(x(8)\right)\right)-0.24\ \{-0.6<x<6.4\}$

25 $2\left(\frac{x+5.56}{3}\right)^2+0.05\left(\cos\left(x(20)\right)\right)-5.73\ \{-1.98<x<-0.591\}$

26 $(x+2.34)^2+(y+2.756)^2=0.15\{x<-1.974\}\{y<-2.63\}$

27 $-3\left(\frac{x-1.82}{3}\right)^2+0.05\left(\cos\left(x(50)\right)\right)+4.22\ \{-2.72<x<-2.1\}$

28 $0.3\left(\frac{x+6}{3}\right)^2+0.05\left(\cos\left(x(8)\right)\right)-1.4\ \{-5.99<x<-2.11\}$

29 $(x+5.9)^2+(y+1.13)^2=0.1\ \{-5.9>x\}$

30 $y=-0.4x+1.06\ \{-5.2<x<-4.41\}$

31 $(x+5.25)^2+(y-3.007)^2=0.02\ \{-5.182>x\}\{y>2.9\}$

32 $0.497(x+4)^2\ +2\{-5.354<x<-4.892\}$

33 $1.5(x+4.14)^2+(y-1.895)^2=1.1\ \{x<-4.607\}\{y<2.396\}$

34 $-0.15\left(\frac{x-1}{3}\right)^2+0.05\left(\cos\left(x(8)\right)\right)+0.02\ \{-5.875<x<-3.5\}$

35 $(x+4.8)^2+(y-0.68)^2=0.15\{x>-4.624\}\{y>0.7\}$

36 $(x+4.272)^2+(y-0.7)^2=0.02\{y<0.703\}\{x<-4.28\}$

37 $(x+3.6)^2+2(y-1.43)^2=2\ \{-4.301<x<-3.9\}\{y<2\}$

38 $(x+3.86)^2+(y-0.6734)^2=0.05\ \{x>-3.899\}\{y<0.55\}$

39 $0.1\sin^{-1}\left(10(x+3.4)\right)-0.2$

40 $-25\left(\frac{x+3.78}{3}\right)^2+0.02\left(\cos\left(x(40)\right)\right)+0.6\ \{-3.67<x<-3.297\}$

41 $y=-0.3x+1.05\ \{-5.5<x<-5.073\}$

42 $(x+5.53)^2+(y-2.562)^2=0.02\ \{x<-5.477\}\{y>2.46\}$

43 $y=-0.9x-2.605\ \{-5.63<x<-4.996\}$

44 $(x+6.498)^2+(y+1.56)^2=0.03\ \{x>-6.337\}$

45 $(x+6.498)^2+(y+1.56)^2=0.03\ \ \{y>-1.4997\}$

46 $y=1.9x+11.155\ \{-7.298<x<-6.66\}$

47 $y=2x+11.05\ \ \{-6.894<x<-6.337\}$

48 $(x+7.25)^2+(y+2.93)^2=0.05\{x<-7.298\}\{y>-2.9\}$

49 $(x+5.2)^2+(y+4.58)^2=8\{x<-7.471\}\{y>-3.9\}$

50 $y=0.07x-2.256\ \{-6.899<x<-6.3\}$

51 $(x+6.3)^2+(y+2.87)^2=0.03\ \{x>-6.3\}\{y>-2.95\}$

52 $(x+6.12)^2+(y+3.12)^2=0.03\{x>-6.146\}\{y>-3.2\}$

53 $(x+6)^2+(y+3.369)^2=0.03\{x>-5.966\}$

54 $y=0.15x-2.64\ \{-6.4<x<-5.938\}$

55 $(x+6.4)^2+(y+3.74)^2=0.02\ \{x<-6.392\}\{y>-3.64\}$

56 $(x+8.69)^2+(y+1.85)^2=8\{y<-3.64\}\{x>-7.1\}$

57 $(x+8.08)^2+(y+3.86)^2=0.02\ \{x>-8.02\}\{y<-3.877\}$

58 $y=0.5x+0.022\ \ \{-12<x<-8.02\}$

59 $y=0.7x+0.781\ \{-12<x<-7.027\}$

Appendix J. Opossum equations

1. $y=-0.1(x-1)^2+6.7\ \{-5.688<x<-4.993\}$
2. $(x+6.03)^2+(y-4.07)^2=2\ \{x>-4.993\}\{y<3.903\}$
3. $1.2(x+2.8)^2+(y-3.896)^2=4\ \{x<-3.727\}\{y>3.9\}$
4. $(x+2.87)^2+(y-4.5)^2=2\ \{y>5.588\}\{x<-2.935\}$
5. $y=0.1(x+2.77)^2+5.91\ \{-2.935<x<-0.723\}$
6. $(x+0.4)^2+(y-5.7)^2=0.5\ \{x<-0.34\}\{y>6.329\}$
7. $y=0.07(x+0.675)^2+6.4\ \{-0.3705<x<2.049\}$
8. $y=-0.08(x-4.5)^2+7.4\ \{2.06<x<9.156\}$
9. $(x-10.6)^2+(y-5.04)^2=0.9\{x<9.991\}\{y<4.939\}$
10. $(x-8.3)^2+(y-4.54)^2=2\ \{x>9.131\}\{y>4.939\}$
11. $y=0.056(x-16.5)^2+1.94\ \{9.991<x<17.41\}$
12. $y=0.0589(x-16)^2+1.5\ \{9.584<x<17.47\}$
13. $(x-17.4)^2+2(y-1.8035)^2=0.067\ \{x>17.394\}\{y>1.6272\}$
14. $y=-0.9x+9.84\ \{8.271<x<9\}$
15. $(x-8.467)^2+(y-2.6)^2=0.08\{x<8.285\}\{y<2.7853\}$
16. $y=-0.2(x-11.581)^2+5\{8.26<x<9.248\}$
17. $(x-9.423)^2+(y-3.74)^2=0.06\ \{y>3.92\}\{x>9.248\}$
18. $(x-8.8)^2+(y-1.495)^2=0.1\ \{x>9\}\{y>1.356\}$
19. $1.3(x-9.813)^2+(y-0.8)^2=1\ \{x<9.084\}\{y>0.532\}$
20. $y=-x+9.5\ \{8.968<x<9.095\}$
21. $(x-9.03)^2+(y-0.329)^2=0.01\ \{y<0.405\}\{x>8.965\}$
22. $y=-x+9.218\ \{8.782<x<8.96\}$
23. $y=-0.1(x-7.4)^2+0.627\ \{7.501<x<8.778\}$
24. $(x-7.5)^2+(y-0.74)^2=0.013\ \{x<7.5\}\{y<0.7989\}$
25. $(x-7.42)^2+(y-0.8866)^2=0.008\ \{x<7.41\}\{y>0.7989\}$
26. $y=-0.07(x-8.7)^2+1.092\ \{7.4<x<8.152\}$
27. $(x-8.15)^2+1.4(y-1.24)^2=0.04\ \{x>8.152\}\{y<1.366\}$
28. $y=-0.6x+6.336\ \{7.04<x<8.28\}$
29. $(x-6.7)^2+(y-1.492)^2=0.5\{x<7.04\}\{y>1.7\}$
30. $y=3x-16.373\ \{5.911<x<6.036\}$
31. $y=2(x-5.183)^2+0.3\ \{5.203<x<5.911\}$
32. $y=0.2x-0.74\ \{4.8<x<5.203\}$
33. $(x-4.79)^2+(y-0.2969)^2=0.006\{x<4.801\}\{y<0.372\}$
34. $y=0.3x-1.0593\{4.771<x<4.95\}$
35. $y=-0.1x+0.9207\ \{4.7<x<4.95\}$
36. $y=0.25x-0.7244\ \{4.36<x<4.7\}$
37. $(x-4.34)^2+(y-0.4404)^2=0.006\ \{x<4.361\}\{y<0.492\}$
38. $(x-4.35)^2+(y-0.53)^2=0.006\ \{x<4.282\}\{y<0.5507\}$
39. $(x-4.31)^2+(y-0.626)^2=0.007\ \{x<4.275\}\{y<0.6969\}$
40. $y=0.7x-2.289\ \{4.266<x<4.5\}$
41. $y=-0.3x+2.21\ \{4.25<x<4.499\}$
42. $(x-4.28)^2+(y-1.0)^2=0.005\ \{x<4.263\}$
43. $y=-0.04x+1.239\ \{4.262<x<4.857\}$
44. $y=0.26x-0.62\{4.282<x<4.6\}$
45. $y=0.5x-1.59\ \{4.274<x<4.6\}$
46. $(x-4.86)^2+(y-1.29)^2=0.06\ \{x>4.86\}\{y<1.218\}$
47. $y=5.5x-26.8\ \{5.095<x<5.239\}$
48. $(x-4.926)^2+(y-2.06)^2=0.1\ \{y>2.0126\}\{x>5.022\}$
49. $y=-0.35x+4.119\ \{3.511<x<5.011\}$
50. $(x-3.45)^2+(y-2.675)^2=0.05\ \{x<3.511\}\{y>2.687\}$
51. $(x-6.36)^2+(y-2.26)^2=10\ \{y<2.687\}\{x<3.28\}$
52. $y=-5(x-3.2)^2+1.58\ \{3.279<x<3.635\}$
53. $(x-3.559)^2+(y-0.62)^2=0.006\ \{y<0.6438\}\{x>3.486\}$
54. $y=-4x+14.537\ \{3.408<x<3.485\}$
55. $y=20x-65\ \{3.275<x<3.292\}$
56. $(x-3.199)^2+(y-0.52)^2=0.006\ \{y<0.5237\}$
57. $(x-4.852)^2+(y-0.6)^2=3\ \{y>0.5088\}\{x<3.14\}$
58. $y=0.5x-0.8\ \{3.292<x<3.408\}$
59. $y=-0.2x+1.49\{2.968<x<3.1399\}$
60. $(x-4.45)^2+(y)^2=3\ \{y>0.615\}\{x<2.968\}$

61 $(x-2.762)^2+(y-0.65)^2=0.006\ \{y<0.6838\}\{x<2.83\}$

62 $(x-4.323)^2+(y-0.1)^2=3\ \{x<2.7972\}\{y>0.6838\}$

63 $y=0.1x+0.64\{2.503<x<2.795\}$

64 $(x-2.5)^2+(y-0.9677)^2=0.006\ \{x<2.5\}$

65 $y=0.1x+0.795\{2.5015<x<2.8529\}$

66 $y=-5(x-2.6)^2+1.4\{2.6375<x<2.8529\}$

67 $(x-2.79)^2+(y-1.67)^2=0.1\{x<2.6375\}\{y<1.5962\}$

68 $y=-5(x-2.135)^2+2.2\ \{2.14<x<2.49\}$

69 $y=-0.27x+2.78\ \{0.074<x<2.158\}$

70 $(x)^2+(y-2.549)^2=0.05\ \{y>2.565\}\{x<0.072\}$

71 $(x+0.67)^2+(y-2.549)^2=0.2\ \{y<2.565\}\{x>-0.534\}$

72 $y=-0.4(x)^2+2.237\ \{-1.746<x<-0.534\}$

73 $y=-0.2(x+1.5)^2+1.03\{-2.1<x<-1.746\}$

74 $(x+2.11)^2+(y-1.035)^2=0.006\ \{x<-2.098\}\{y<1.1079\}$

75 $y=0.5x+2.176\{-2.1362<x<-1.9761\}$

76 $y=0.1(x+2.4)^2+1.17\ \{-2.4<x<-1.977\}$

77 $y=-0.25(x+2.4)^2+1.17\ \{-2.886<x<-2.4\}$

78 $(x+2.9)^2+(y-1.187)^2=0.006\ \{x<-2.8879\}\{y<1.26\}$

79 $y=-0.3(x+2.4)^2+1.343\{-2.925<x<-2.6\}$

80 $y=-0.2(x+2.9)^2+1.349\ \{-3.1<x<-2.6\}$

81 $(x+3.1)^2+(y-1.4188)^2=0.006\ \{x<-3.1\}$

82 $y=-0.3(x+2.7)^2+1.546\ \{-3.1096<x<-2.8716\}$

83 $y=-0.1x+1.25\{-2.9479<x<-2.8716\}$

84 $(x+2.95)^2+(y-1.608)^2=0.004\ \{x<-2.9479\}\{y<1.6704\}$

85 $y=-0.1(x+2.3)^2+1.714\ \{-2.96<x<-2.128\}$

86 $(x+2.1)^2+(y-2.026)^2=0.1\ \{x>-2.12\}\{y<1.8694\}$

87 $y=0.63(x+3)^2+1\ \{-1.273>x>-1.825\}$

88 $(x+1.627)^2+(y-3)^2=0.14\ \{y>2.8073\}\{x>-1.627\}$

89 $y=0.101(x+6)^2+1.966\ \{-5.561<x<-2.7\}$

90 $(x+5.53)^2+(y-2.156)^2=0.03\{y<2.275\}\{x<-5.51\}$

91 $y=-0.4(x+1.8)^2+3.39\ \{-2.702<x<-1.627\}$

92 $(x+2.815)^2+(y-6.3)^2=1\ \{x<-3.616\}\{y<6.498\}$

93 $1.3(x+3.567)^2+(y-6.45)^2=0.07\ \{y>6.5\}\{x<-3.37\}$

94 $(x+5.197)^2+(y-5.3)^2=5\ \{x>-3.360\}\{y>5.903\}$

95 $(x+2.4)^2+(y-5.1)^2=0.3\{y<5.392\}\{y>5\}$

96 $y=-1.9(x+2.4)^2+5.8\ \{y>5.392\}$

97 $y=-(x+0.862)^2+6\ \{-2<x<-1.8612\}$