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Short-Term Volatility Curve Predictions Using Singular Spectrum Analysis

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Abstract

This project aims to produce accurate volatility forecasts, using high-frequency financial time series data. The primary mathematical methods used are functional data analysis (statistical analysis on smoothed curve observations), time series analysis techniques such as autoregressive models, and a comparison between multivariate and univariate singular spectrum analysis. These results aim to be useful for financial risk quantification.

Introduction

Quantifying financial risks for high-frequency financial time series data plays an important role in portfolio optimization and options pricing[1]. It can be achieved by analyzing volatility curves, which approximately represent the absolute percentage differences in prices between subsequent data points.

Research Goals

Primary Research Goals: To come up with:

1. Accurate short-term volatility curve forecasts (1- to 5-day ahead forecasts), and;
2. Reliable prediction bands.

Models Being Compared: Two novel volatility models which utilize singular spectrum analysis (SSA), a recent time series technique, to extract trends and seasonalities in volatility curves[2]. Specifically,

(1) Univariate SSA (USSA) and (2) Multivariate SSA (MSSA)

are compared using the mean-square error (MSE) values of appropriately transformed predicted volatility curves. A lower MSE value indicates a better forecast performance.

Data

Stocks Analyzed: Apple, BP, and Pfizer.

Time Resolution and Period: One-minute intraday price data from January 2010 to January 2018.

Data Source: QuantQuote (https://quantquote.com/).

Modeling Approach

Our novel SSA-based volatility curve models are constructed using the following ideas:

1. Symmetrizing Data Transformations

To make the distribution of the volatility curve data symmetric, we have applied ¼ power, mean-standardization, log transformation, and Yeo-Johnson transformations[3]. After these transformations, we have a unit-free, symmetric curves that can be used to produce accurate forecasts.

2. Smoothing the Transformed Volatility Curves

The transformed volatility curve data are not suitable for statistical analysis due to their roughness. They are smoothed using 15 Fourier basis functions. After that, about a dozen of empirical functional principal components (EFPCs) are used to capture $\geq 90\%$ of the variation in the smoothed data.

3. Capturing Deterministic and Stochastic Components

SSA is applied to the scores corresponding to the EFPCs for modeling their deterministic components. Because the EFPCs may be viewed as multivariate time series, we considered both the univariate and multivariate SSA approaches. After removing the deterministic components, an autoregressive (AR) model, which predicts future observations of stationary time series based on a number of previous observations, is applied to describe the stochastic components.

4. Forecasting Using SSA and AR Predictions

Predicted values from the SSA and AR models are combined to generate transformed and smoothed volatility curve forecasts. They are then back-transformed to generate volatility curve forecasts. By comparing these forecasts to the past actual curves, prediction bands are also constructed by taking appropriate percentiles of their daily maximum errors (see Fig. 1).

Analysis

The predicted curves correctly capture a typical diurnal pattern (higher volatility at the beginning and end of day; see Fig. 1). The MSE values of USSA are about one fourth of that of MSSA. That indicates that USSA tends to produce much more accurate short-term volatility curve forecasts than MSSA (see Fig. 2).

Volatility Curve Forecasts and Prediction Bands

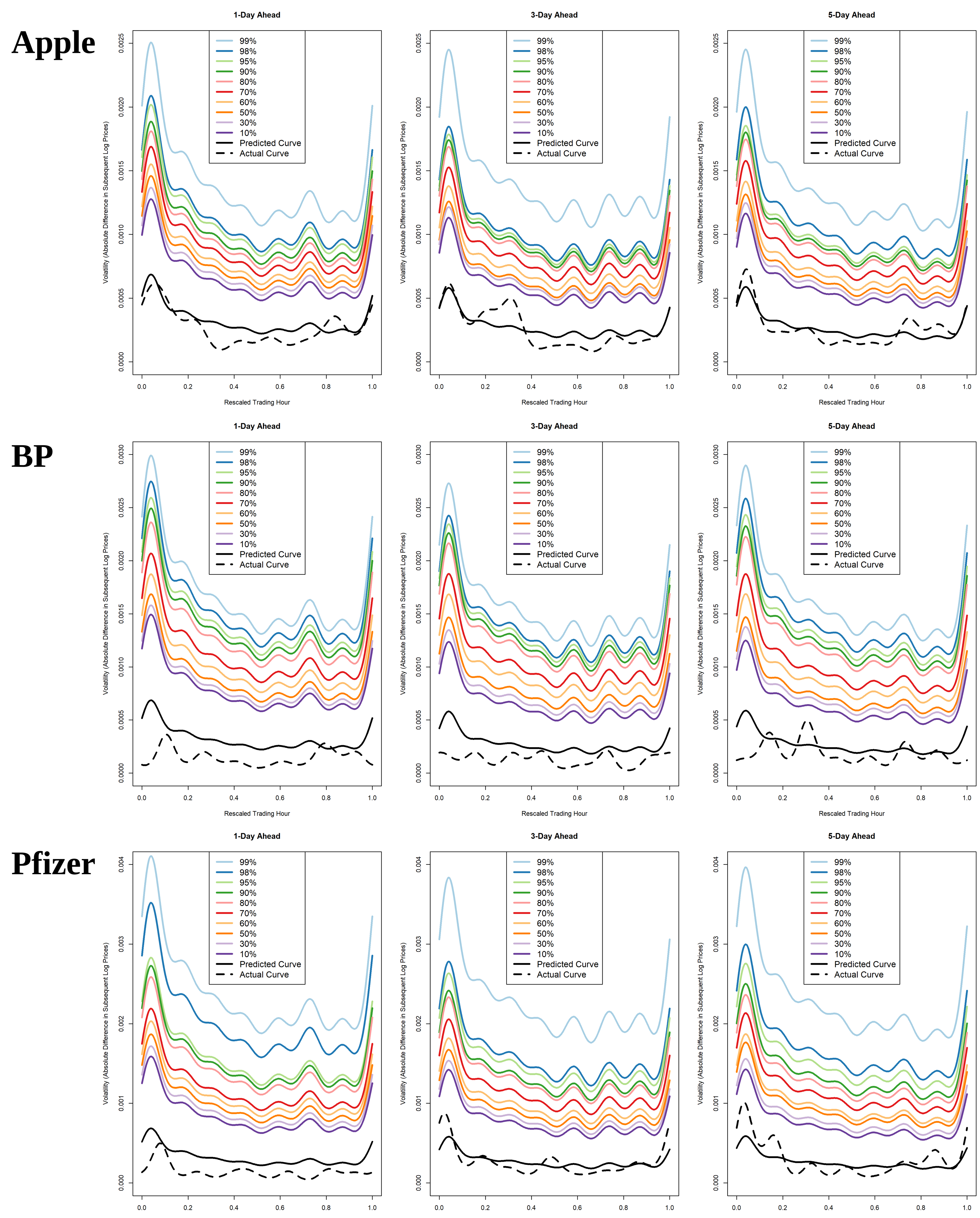

Fig. 1. One-sided 1-, 3-, and 5-day ahead prediction bands at various confidence levels and predicted curves of the USSA model for Apple, BP, and Pfizer on January 27, 29, and 31, 2018.

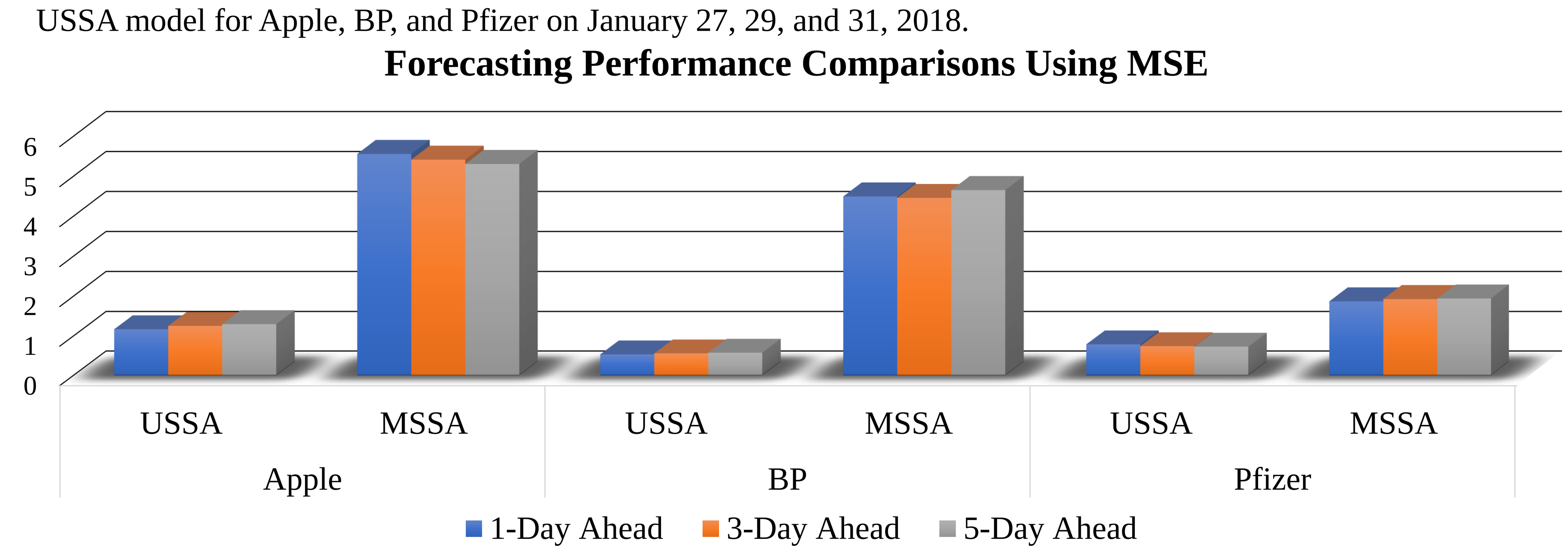

Fig. 2. The mean-square error (MSE) values of the USSA and MSSA models for Apple, BP, and Pfizer. Their values, which are on a scale of 10^{-8}, are calculated by comparing 250 predicted curves to the actual curves.

Conclusions and Future Research

We have discovered that USSA tends to produce much more accurate short-term volatility curve forecasts than MSSA based on Apple, BP, and Pfizer 1-minute intraday stock price data. Moreover, the predicted curves and their prediction intervals seem to appropriately capture the diurnal pattern and upper bounds of the smooth volatility curves. We are planning to examine more stock price data from different industries to strengthen our conclusion.

Acknowledgement

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