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Not Not Double Negation

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According to the law of the excluded middle (hereafter 'LEM') any sentence of the form 'p or not p' is logically true. In other words, no matter how the world is like in itself, any sentence of this form must be true. Yet the truth of this theorem remains highly controversial. For it appears to be subject to counterexamples. On the other hand, according to the law of non-contradiction, no sentence of the form 'p and not p' is true. The law of non-contradiction is an uncontroversial theorem in logic. Yet a simple proof allows us to derive the former from the latter using the rule of double negation. According to this rule, we can infer from not not p that p. The result is that the rule of double negation has been rejected in logical systems that reject LEM. Even with these rejections, double negation seems obviously true.

In this paper I discuss the concepts of logic, LEM, double negation, and the law of non-contradiction. Following this are a few reasons for rejecting LEM and double negation with discussion of costs associated with rejecting both. The point of this paper is not to convince you of one side over the other, but rather to allow you to make that choice according to the presented material.

WHAT IS LOGIC?

We as humans want to know what is true and what is false. We want to reason well about any topic, which doesn't just include knowing what is true and false but infer correctly from what we believe. This is what logic is all about.

Philosophical logic can be traced back to the time of Aristotle or even prior and through the years different branches of logic have arisen. All branches of logic share the same purpose, which is to use principles and methods to determine whether an argument (a set of statements where some statements, premises, support another, conclusion) or statement is valid or sound. A valid argument is one that preserves truth from premises to conclusion.

In other words, if the premises of an argument are true then the conclusion is guaranteed to be true as well. A more formal definition of logic can be given as logic is the study of methods for evaluating whether the premises of an argument adequately support its conclusion. Logic doesn't just tell us whether an argument is valid or invalid, but it gives us methods to allow us to test the validity of different arguments and statements. Those methods are topic neutral, which means that it doesn't matter if the argument or statement is about people or aliens, the significance lies in the logical form of the argument. The English arguments are turned into schematic arguments with variables without disruption of the logical structure of the argument.

The Law of the Excluded Middle

A fundamental principle of classical logic is LEM. LEM states that for any statement p , either p is true or not p is true, that is p or not p . If this principle states that every statement is true, or its negation is true then it follows that there is no third possibility or middle ground. For any given statement, p or not p is true. There's no case where a statement wouldn't follow this principle. For example, it is raining, or it is not raining. LEM states that either it is raining, or it is not raining. It can't be that both are true and that both are false. It's one or the other.

Double Negation

LEM leads us to equivalence rules. Some logical expressions have multiple identical representations, so they can be written one way and the same logical expression can be written in a different way, and still maintain its same meaning. These equivalence rules are used to make things simpler when figuring out if an argument is valid. For example, in mathematics saying $2 + 2 + 2$ is the same as saying 2×3 . These expressions are equivalent to each other as they bring out the same solution. Double negation is one of these equivalence rules and it's sought to be the simplest, but maybe the most controversial one.

Double negation states that p is true if and only if not not p is true. Meaning p is equivalent to not not p and vice versa. Something being not not is the same as saying that the statement is not a negation, so it becomes the same as a non-negated statement. It is not not raining is the same as saying it is raining.

Law of Non-Contradiction

The uncontroversial theorem in logic, law of non-contradiction, allows us to derive LEM from it using double negation. The law of non-contradiction states that contradictory statements or propositions cannot be both true. A statement cannot be both true and false. It can't be that it is both raining and not raining. This theorem is vital to attaining knowledge and understanding reality because without it we could say that nothing really has a truth value. We could easily undermine biology, mathematics, and proven facts. Imagine a world without the law of non-contradiction. We could say that I exist, and I don't exist. I am human and I am not human. I have a heart and I don't have a heart. $1 + 1$ equals 2 and $1 + 1$ doesn't equal 2. Without the law of non-contradiction everything we know is undetermined.

A simple proof can show how LEM and double negation relate to each other with the law of non-contradiction. The law of non-contradiction would state not (p and not p). This is saying that it's not the case that both p and not p are true. If that's the case, we get not p or not not p . It follows that either not p is true or not not p is true. Using our double negation rule, we derive LEM to get not p or p . This means that if it can't be that both p and not p are true then it must be that either p or not p is true.

Not only does the proof show how LEM and double negation are connected, but it provides an argument to accept both LEM and double negation. If the law of non-contradiction is valid and uncontroversial, we must accept LEM and double negation.

THE DEBATE OF LEM AND DOUBLE NEGATION

The arguments against double negation stem from contradicting LEM. The arguments that I will begin with will be against LEM, but they tend to be followed by rejection of double negation. It's important to keep in mind that if LEM is successfully rejected then it must be that double negation is rejected as well. If LEM is accepted, then we should be inclined to accept double negation.

Indeterminacy

Indeterminacy is a popular contradiction to LEM, which says there is no fact of the matter. LEM claims there is no gray area or middle ground. There is only true or false, but indeterminacy says there are not only two choices to everything. Some things are indeterminate and there is no one answer to them. For example, there is no fact of the matter of how many mountains exist as there are mountains, but no definite answer to how many. Indeterminacy raises questions like what minimum worth suffices for being rich, what is considered as bald, or who is better MJ or LeBron? There is no one answer to these, so these questions as they are indeterminate.

Indeterminacy also explains how LEM goes against free will. We as humans like to believe that we have free will or at least have free will for a majority of things we do. We like to think we have a choice in what we want to eat for dinner, whether we decide to go to college or not, or whether we take action for a particular thing. Some may believe that things are pre-determined, but we still make choices which we define as free will. If we didn't believe in free will then we would just be doing nothing and let destiny do its thing. Indeterminacy states that LEM does exactly this.

We have the answers to the past and can assign truth values to them, but how about the future? Imagine you have a final coming up and we know that either you will get an A, or you won't. Either one of these outcomes will come about, but in the present it's indeterminate which one. Till you take the final and get the grade back, you won't know whether you got an A or didn't. In other words, it's indeterminate whether you will get an A or not get an A (p or not p). If LEM tells us that things are determinate, so you will get an A or not an A, then why study for the final? One of these are true and it's determinate, so we can't possibly change the outcome. Indeterminacy says that's obviously false. In this sense, LEM is saying that free will is not a thing, but we like to believe it is. If we consider LEM and free will not to be true, but rather it's true that it's indeterminate whether you will get an A or not an A, then you can work for an A. You should be able to study to make sure your outcome is an A.

Vagueness

Vagueness is a form of indeterminacy, which questions at what point can we say something is true or false. For example, at what point can it be said whether Bob is bald or not bald, at what point is Harry considered tall compared to being considered not tall, or at what point is Rob considered rich compared to not rich? This may seem like the indeterminacy examples, but rather than saying that things are indeterminate, these show that there is vagueness in when something suffices as true or false. In other words, there is vagueness in the words we try to define as true or false. There is vagueness in the word bald, tall, and rich.

An uncomplicated response to indeterminacy and vagueness arguments is that these things are just intuition. There is really no knowable fact of the matter, and we are just able to easily conclude when someone is bald or rich. Knowing that Rob makes a million annually we would agree to say that it is true that he is rich. Would million dollar making Rob losing a dollar

make him poor? The inclined answer is no. If Harry is 5 foot and grows one inch, then would Harry be considered tall? Another no. Our intuition can answer these questions with little hesitation. How about in more extreme cases? If 5-foot Harry grows 6 inches, is he then considered tall? Well, he clearly grew a significant amount so according to his previous height he could be considered tall but thinking of 5 foot 6 not relative to Harry might be considered as short. If we can come up with counterexamples, then this response to indeterminacy and vagueness is not so strong.

A stronger example is simply that indeterminacy and vagueness do not threaten LEM. Imagine bald Bob. Bob is on an operating table having one of his arms reattached. At the present stage of the procedure, it's vague whether the arm is reattached or not. The surgeon asks the resident, "what's the current weight?" Bob's body weight is 150 without the arm and 170 with the arm. The resident then responds, "it's vague whether he weighs 150 or 170, but it is clear that he weighs one of these." It's vague whether Bob weighs 150 and it's vague whether Bob weighs 170. Intuitively, Bob clearly weighs either 150 or 170. In this instance LEM is still true even if neither are clear. We may not have the answer, but it is true that Bob either weighs 150 or doesn't weigh 150.

The case of bald Bob and any argument it's true that there may be vagueness and indeterminacy to whether p or not p is true. It may be indeterminate whether p is true or not p is true, but LEM doesn't claim that it needs to be determinate. All LEM states is that either p or not p . It's the case that either it is true or not true even if we don't have the answer. According to LEM, Bob is either bald or not bald, but it's vague whether he is or not. What does it take to make it true that Bob is bald than for him to be bald and what more could it take for him to be bald than for it to be true that he is bald? It is vague which, either true or false that Bob is bald,

but that means it's true to say that Bob is bald or not bald. The statement itself is true and the vagueness lies in the choice. LEM is saved as both p or not p are present and which one out of these two is what comes into question.

Double Negation in English

Indeterminacy also shows up in double negation when used in English grammar and conversation. Double negatives in English grammar are usually frowned upon, but when used what does it mean? Consider Bob and Rob are having a conversation and Bob asks Rob, "do you want to go to the party tonight?" Rob replies "I don't not want to go." What is Rob really trying to say? We tend to annotate this by saying that it's neither true Rob doesn't want to go and that he does want to go. It does seem like he is more inclined to wanting to go. There's an indeterminacy factor in this answer but consider an example in the court room. The attorney asks Rob, "did you kill Bob?" Rob answers "I didn't not kill Bob." What does double negation mean in this circumstance? It's highly unlikely for the jury to say that it's indeterminate if he did it or not because of his use of double negation. It's clear that Rob did kill Bob if he is negating the negation. Maybe this is just a case-by-case phenomenon.

CONCLUSION

In this paper I discussed the basics of logic, LEM, and double negation. I then built a connection of LEM and double negation through the law of non-contradiction. This was followed by arguments of indeterminacy and vagueness against LEM and responses to these arguments. In addition, we looked at double negation in English and how that plays a role with indeterminacy.

If the indeterminacy and vagueness arguments against LEM are successful, then double negation is rejected according to the law of non-contradiction. Without LEM it's hard to uphold

double negation and without double negation it's hard to uphold LEM. If the indeterminacy and vagueness arguments are unsuccessful, then we cannot reject either LEM or double negation. Is it successful or is it not? This paper has laid out the fundamentals with arguments followed by responses to these arguments. It's up to the reader to decide which side is convincing, but as a writer it's hard for me to reject LEM or double negation. LEM doesn't need to be linked to determinacy or indeterminacy. It just states that either something is true, or it is not true. Both LEM and double negation are fundamental principles of logic that act as a foundation of the methods known and used in logic.

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