Spring 2016

Curriculum Development

Courtney Knox
Western Washington University

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Curriculum Development
By Courtney Knox
Honors Senior Project
2014-2016
This project examined the process of developing curriculum and analyzed its efficacy. Initially there were two sets of curriculum involved, but only one is included here because I was unable to gain the permission to publicly publish the other. The curriculum that is included is the curriculum I developed the Math Club at Fairhaven Middle School in Bellingham, WA. This curriculum was developed and implemented independently of any official curriculum regulations. Thus, I was free to personalize and change the curriculum based on what I saw was most needed in the classroom setting. However, there were still challenges that needed to be overcome in order to reach the goals that I set for the curriculum.

I was given the position of Math Club Teacher for Fairhaven Middle School (FMS) in November of 2014. When I first started out, I had little to no direction in regards to what I was supposed to be teaching. So, first I developed the following three goals: I wanted the FMS Math club to… (1) be a welcoming environment where students could explore mathematics, (2) provide challenging material to stimulate problem solving, and (3) prepare students to compete in the Whatcom County Math Championship. So, to begin I studied the competition tests from previous years. After extensive study I developed a list of topics and skills that my students would be expected to know and developed worksheets and lesson plans based around them.

Included in this document is the “FMS Math Club Workbook” and the “FMS Math Club Answer Book.” The Workbook contains all of the material that I developed to help my students learn the necessary skills and topics. The problems in the Workbook are gathered from a variety of sources including various internet sites, the MOEMS Contest Problem book (Volume 5), and Edward Zaccaro’s Challenge Math book. I also put in a few problems from my own imagination. I do not take credit for all of the problems though, and make note of that on the back cover of the Workbook. The Answer Book on the other hand is entirely my creation. It includes detailed answer keys for everything included in the Workbook.

The Answer Book is the bulk of the curriculum that I developed. Through it, not only was I able to help the students be metacognitive, but I also was able to teach them the reasoning behind the various topics and skills. I didn’t just want them to memorize a formula, I wanted them to know why something mathematical was the way it was and how it worked inside and out. Through my Answer Book I was able to show them my thinking, my logic, and explain different ways of solving the various problems.

After the execution of the curriculum, surveys were given to the students and parents to analyze the curricula and establish changes that could be made in the future to make the curricula more effective. Overall, my curriculum was very effective. At the 2016 Whatcom County Math Championship, nine of the ten teams from FMS placed in 5th place or higher and two individual students got first in their grade level. Additionally, during the two years that I taught Math Club I expanded the program from 20 students to 40 students, providing extra challenge for those who weren’t being challenged in their regular classroom and extra exposure for those who needed to sharpen their skills.
Fairhaven Middle School
Math Club
2015-2016
If found, please return this book to:

Name: __________________________________
Grade: __________________________________
School: _______ Fairhaven Middle School____
Phone Number: ___________________________
Email: __________________________________
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Notes
Useful Formulas
Useful Formulas

Area: $WL$
Perimeter: $2(W + L)$

Area: $\pi r^2$
Circumference: $2\pi r$ OR $d\pi$
Arc length: $\alpha r$

Area: $(1/2)bh$
Perimeter: $s_1 + s_2 + b$

If the $\Delta$ is equilateral (meaning, $s_1 = s_2 = b$)
Area: $((\sqrt{3}/4)(b^2)$
Perimeter: $b^3$

For right $\Delta$’s, to find the length of a side use the Pythagorean Theorem: $a^2 + b^2 = c^2$
Where ‘c’ is the length of the hypotenuse and ‘a’ and ‘b’ are the lengths of the two other sides.

Area: $(1/2)(b_1 + b_2)h$
Perimeter: $s_1 + b_1 + s_2 + b_2$
Rectangular Prism
Volume: $LWH$

Cylinder
Volume: $\pi r^2h$

Sphere
Volume: $(4/3)\pi r^3$

Combination Formula:
$$\frac{n!}{r!(n-r)!}$$

Permutation Formula:
$$\frac{n!}{(n-r)!}$$
Quick Notes on Permutations with Repetition

Maybe some of you have already noticed this, but, in case you haven’t, when you have $n$ things and you are “permuting” $r$ of them and $n = r$ you get the following:

Permutation Formula: $\frac{n!}{(n-r)!} = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$

Now, that’s all well and good until you are trying to permute a collection of things that may include repeated items. Let’s say you are given four letters: O, M, N, O. How many different arrangements (permutations) can you make?

<table>
<thead>
<tr>
<th>MOON</th>
<th>MONO</th>
<th>MNOO</th>
</tr>
</thead>
<tbody>
<tr>
<td>OMON</td>
<td>OMNO</td>
<td>OOMN</td>
</tr>
<tr>
<td>OONM</td>
<td>ONMO</td>
<td>ONOM</td>
</tr>
<tr>
<td>NOOM</td>
<td>NOMO</td>
<td>NMOO</td>
</tr>
</tbody>
</table>

Why are there only 12, instead of $4! = 24$ ways? You can't tell the "O"s apart, so their order doesn't matter. Suppose one of the "O"s was capital and the other one was small. Then you would have "MOoN" and "MoON". But our "O"s are alike and we can't tell the difference so we have "MOON" and "MOON", which are the same. We count them once. Since there are 2 "O"s, there are $2! = 2 \times 1 = 2$ ways of ordering them. So the number of permutations of letters in the word MOON is:

$$\frac{4!}{2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1} = 4 \times 3 = 12 \text{ ways}$$

So, in short, when you have $n$ things, you are “permuting” $r$ of them, $n = r$, and $p$ of them are the same….

Permutation Formula: $\frac{n!}{p!}$
Converting to and from Base 10

Converting from Base 10 (decimal form) to Base ∗

\[
\begin{array}{c|cccc}
\text{value} & \cdots & \alpha^3 & \alpha^2 & \alpha^1 & \alpha^0 \\
\hline
\text{base}^\text{power} & \cdots & \alpha & \beta & \gamma & \delta \\
\end{array}
\]

\[
\begin{align*}
194 / \alpha &= 0 \text{ remainder } 194 \\
194 / \beta &= y \text{ remainder } 14 \\
14 / \gamma &= x \text{ remainder } 2 \\
2 / \delta &= w \text{ remainder } 0
\end{align*}
\]

assuming \( y, x, \) and \( w \) are all integers, \( 194_∗ = 0y\times\delta_0 = y\times\delta_{10} \)

Converting to Base 10 (decimal form) from Base ∗

\[
\begin{array}{c|ccc}
\text{corresponding base}^\text{power} & 2 & 5 & 7 \\
\hline
\text{value} & \star^2 & \star^1 & \star^0 \\
\end{array}
\]

\[
\begin{align*}
2 & \rightarrow 2 \times \star^2 = \lambda \\
5 & \rightarrow 5 \times \star^1 = \mu \\
7 & \rightarrow 7 \times \star^0 = \psi
\end{align*}
\]

\( 257_∗ = 7_0 \)

\( \equiv \) means "equivalent to"
Worksheets
Area and Perimeter

Use 3.14 as an approximation for $\pi$

1. A circle rolls, once, without slipping, along the outside of a square with side of length 4 inches, and returns to its starting point at $A$. The radius of the circle is 1 inch. To the nearest hundredth of an inch, how far does the center of the circle travel?

2. A rectangle is divided into four smaller rectangles whose areas in sq cm are 35, 42, 10, and $N$, as shown. The length of each side of every rectangle is a whole number, What is the value of $N$, in sq cm?

3. As shown, each of the four congruent circles just touches two other circles and two sides of the outer square. The centers of the four circles are connected to form the inner square. If the area of the outer square is 100 sq cm, what is the area of the inner square, in sq cm?

4. Segment $EF$ divides rectangle $ABCD$ into square $I$ and rectangle $II$. The area of rectangle $ABCD$ is 144 sq cm. The area of rectangle $II$ is three times the area of square $I$. Find the perimeter of rectangle $II$, in cm.

5. Two runners are racing 4 times around a circular track with a circumference of 1000 feet. The first runner will run on the edge of the track, while the second runner will run 3 feet outside the edge of the track. How much more distance will the second runner cover than the first runner?

6. What is the perimeter of this figure?

7. The area of the outside square is 16 square inches. Points A, B, C, and D are midpoints on their respective lines. What is the diameter of the shaded circle?

8. Kristin needed to paint the floor of her circular room. She started working from the outside and gradually moved toward the center of the room. She has used a gallon of paint and has painted only a 2 foot wide strip around the outside of her room. If the diameter of the entire room is 20 feet, how many more gallons of paint will she need to buy? (Paint can only be purchase in full gallons.)

9. What is the area of the shaded part if the diameter of the circle is 10 feet?

10. A recipe will make 10 pancakes that are each 8 inches in diameter. If you decided to make 2 inch diameter pancakes, how many of the smaller pancakes would this recipe make?
Volume

Use 3.14 as an approximation for $\pi$

1. How many cubic inches are in a cubic foot?

2. A waterfall has a flow rate of 2700 cubic feet of water per second. How many cubic yards of water go over the waterfall in a year?

3. The moon and the earth are almost perfect spheres. The moon is 2000 miles in diameter compared to the earth’s 8000 mile diameter. The diameter of the earth is 4 times larger that of the moon. The volume of the earth is how many times larger than the moon’s volume?

4. A rectangular prism has faces with the following surface areas: 108 in$^2$, 144 in$^2$ and 192 in$^2$. What is the volume of the prism, in cubic inches?

5. How many cubic yards are in a cubic mile?

6. Three cubes are dropped into a container of water that is filled to the top. They are a 1 inch cube, a 1 foot cube, and a 1 yard cube. What is the volume of the water that spills out?

7. Gold can be pounded into very thin sheets. If a 1 cm. cube is pounded into a sheet exactly 1.25 cm. wide and 20 cm. long, how thick is the sheet?

8. What is the volume of 40 washers that are each 1/8 inch thick with 4 inch outside diameters and 2 inch inside diameters?
Decimals, Fractions, and Percents

1. For parts a, b, c, and d, answer the following questions in regards to the given numbers x and y:
   - What is \( \frac{x}{y} \) in reduced fraction form?
   - What is \( \frac{x^2}{y^2} \) in decimal form?
   - What is 34% of x?
   - What is the remainder of \( x + y \)?
     a. x = 20, y = 19
     b. x = 63, y = 48
     c. x = 378, y = 105
     d. x = 216, y = 288

2. Kristin’s car goes 18.95 miles for every gallon of gas. If she is planning a trip of 379 miles and gas costs $1.23 per gallon, what is the cost of gas for the trip?

3. What is the value of N?

\[ \frac{1}{5} = \frac{1}{6} + \frac{1}{N} \]

4. Last week Jessica beat Emily at checkers in 20% of their matches. Emily won 12 matches. How many matches did Jessica win?

5. Express the extended fraction shown at the right as a simple fraction in lowest terms.

\[ \frac{3}{4} + \frac{3}{4} \]

6. Chantelle is going on an 8-day hike of 82.5 miles. She plans to hike 8.25 hours every day for each of those 8 days. How many miles will she cover each hour of her trip?

7. Express as a single fraction in lowest terms:

\[ \frac{7}{19} \times \frac{13}{44} + \frac{7}{19} \times \frac{19}{44} + \frac{7}{19} \times \frac{25}{44} + \frac{7}{19} \times \frac{31}{44} \]

8. A 5” X 7” picture is enlarged to 8” X 10”. By what percent did it’s area increase?

9. Jenny and Lenny pick a forth and a third of a treeful of apples, respectively. Penny picks the rest of the apples. If Lenny picks 7 more apples than Jenny does, how many apples does Penny pick?

10. 75% of 76 is the same as 95% of what whole number?
Mean, Median, Mode, and Range

1. For every set of numbers below, find the mean, median, mode, and range:
   a. 12, 13, 14, 15, 17, 18, 19
   b. 1.1, 1, 1.01, 1, 1.001, 1, 1.0001
   c. 0, 0, 2, 2, 4, 4, 6, 6, 7
   d. π, √9, 3.125, 3, 3.22, 3.3, 3!

2. Jenn has taken four math exams over the course of the semester. Her scores were an 85.5, a 65.75, a 92.5, and a 91.6. What is her average exam score for the semester?

3. What is the mean (average) of all the counting numbers less than 100 which are multiples of 3?

4. Chad recently launched a new website. In the past six days, he has recorded the following number of daily hits: 37, 29, 37, 56, 45, 38. He is hoping at week’s end to have an average number of 40 hits per day. To achieve this, how many hits must he have on the final day of the week?

5. What is the arithmetic mean of all the positive two-digit multiples of 4?

6. On a recent Geography test, Amanda scored 5 points below the class average, Barb scored 8 points above the class average, and Colin scored an 82. The average of the scores of Amanda, Barb, and Colin was equal to the class average. What was the class average?

7. Brian was comparison shopping for DVD players. He decided he wanted to purchase a DVD player that was in the middle of the price ranges. The prices he was quoted include the following: $59.99, $219.99, $79.99, $84.99, $159.99, $109.99, $35.99. Which DVD player did Brian select?

8. The average of five numbers is 25. Four of the numbers are 19, 21, 24, and 25. Find the other number.

9. The mean, the median, and the mode of the five numbers below are all equal. What number does A represent?
   1.8  1.6  2.1  1.7  A

10. The team scored the following number of runs in their games this season: 6, 2, 5, 9, 11, 4, 5, 8, 6, 7, 5. If they want to end the season with an average of 6 runs per game, how many runs must they score in their final game of the season?
1. The sum of two prime numbers is 2001. What is the larger of these primes?

2. The Fibonacci sequence begins 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, and so on. How many of the first thirty terms of this sequence are odd numbers?

3. Suppose \( P \) and \( Q \) both represent prime numbers such that \( 5 \times P + 7 \times Q = 109 \). Find the value of the prime \( P \).

4. Sarah used 9 congruent “unit” triangles to form a large triangle of 3 rows, with no gaps and no overlaps (shown). How many unit triangles does Courtney need to form a similar large triangle of 12 rows in the same way?

5. What whole number between 100 and 200 is both a perfect square and a multiple of 7?

6. 14 can be expressed as the sum of two prime numbers in exactly two different ways: \( 11 + 3 = 14 \) and \( 7 + 7 = 14 \). In how many ways can 40 be expressed as the sum of two prime numbers?

7. How many whole numbers between 1 and 150 have exactly three different factors?

8. The sequence 2, 3, 5, 6, 7, 10, … consists of all counting numbers which are neither perfect squares nor perfect cubes. Find the 75th term of this sequence.

9. List all the counting numbers less than 50 that have exactly three factors.

10. What is the sum of all the prime numbers between 50 and 60?
Probability

1. If you flip a coin 7 times, what is the probability that you will get heads on every flip?

2. If you are planning to roll two dice a thousand times. How many of those rolls would you expect to have a sum equal 7 when the numbers on the two dice are added together?

3. If you pick on card from a deck, what is the probability that the card will be a diamond or the ace of spades? (Note: a standard card deck has 52 cards in it).

4. Eric and Mike are playing a game with coins and dice. Eric is tossing three coins in the air. If he gets three heads, he is the winner. Mike is rolling two dice. If he rolls a seven, he wins. Is this a fair game? Meaning, is the probability that each boy will win equal?

5. In a jar of quarters, nickels and pennies, the value of the quarters is $10. The value of the nickels is $10, and the value of the pennies is also $10. If you picked one coin, what is the probability of getting a penny?

6. If the pages of a book are numbered consecutively from 1 through 177, inclusive. If a page is chosen at random, what is the probability that the page number will contain the digit ‘1’?

7. A whole number is chosen at random from the integers 11, 12, 13, …, 49, 50. What is the probability that the number is divisible by 3 or by 5?

8. When a state has a lottery, it usually has a player pick five numbers from the sequence 1-49 (inclusive). If the numbers drawn match the player’s chosen numbers, in any order, then he would win a substantial amount of money. What is the probability of winning this type of lottery?

9. If you pick one card from each of 5 different decks, what is the probability that you will pick at least one spade?

10. A young couple decided to have six children. They did not care if their children were boys or girls. The only wish that they had was that they would have at least one girl and at least one boy. What is the probability that this couple, after having children, would have at least one boy and at least one girl?

11. If you pick three cards from a deck of cards, what is the probability that you will get three of a kind?

12. If you pick three cards from a deck of cards, what is the probability that you will pick three aces?
Algebra

1. Eight times a number is the same as that number plus 84. What is the number?

2. If $3 \times ☼ - 25 = 8$, then find the value of $3 \times ☼ + 25$.

3. In a pile of nickels and dimes, the number of nickels is three times the number of dimes and the pile is worth $1.75. How many nickels are there?

4. Carol chooses a number. She multiplies it by 4, then adds 8, then divides by 4, and finally subtracts 8. Her end result is 4. What number did she choose?

5. Luke has $21$ more than Rachel and $48$ more than Daniel. All together they have $168$. How much money does Luke have?

6. The sum of 7 consecutive integers is 105. What is the sum of the least and greatest of these integers?

7. Suppose that $a ▽ b$ means $a + a - b$. For example, $3 ▽ 4$ means $3 + 3 - 4$, which is another name for 2. If $4 ▽ 5$ and $6 ▽ □$ represent the same number, what is the value of □?

8. Ari, Barry, Cara, Dara, and Erin have a total of $85$. Ari and Barry have a total of $40$. Cara and Dara have a total of $30$. Erin and Ari have a total of $20$. How many dollars does Barry have?

9. 9 apes weigh as much as 4 bears. 8 bears weigh as much as 15 cougars. 10 cougars weigh as much as 27 deer. How many deer weigh the same as 4 apes?

10. The cafeteria sells each apple at one price and each banana at another price. For 5 apples and 3 bananas Dan pays $5.70. For 3 apples and 5 bananas Chris pays $4.70. The price of one apple is how much more than the price of one banana, in cents?

11. There are 5 people standing in a room. Rick is 5 times the age of Mike who is half the age of Larry. Ed is 30 years younger than double Larry and Mike’s combined ages. Daniel is 79 years younger than Rick. The sum of their ages is 271. How old is Daniel?

12. A telephone call costs 25 cents for the first three minutes and 3 cents for each additional minute. If Jason pays 40 cents for a call, for how many minutes does the call last?

13. Together, Juan and Maria have 72 marbles. Juan gives Maria half his marbles and then 12 more marbles. Maria now has three time as many marbles as Juan has. How many marbles did Maria have originally?

14. Curt mistakenly multiplied a positive number by 10, when he should have divided the original positive number by 10. The answer he found was 33.66 more than the answer he should have found. Find the original positive number.

15. All members of the Math Club paid the same amount for their End-Of-Year Party. The boys paid a total of $90$ and the girls paid a total of $60$. However, buying gifts raised the totally cost to $210$, so each member paid an additional $2$. How many members are boys?
1. Points $A$, $B$, $C$, $D$, and $E$ are points of the circle as shown. How many different triangles can be formed by joining any three of these points? *(Note: the triangle formed by joining $A$ to $B$ to $C$ is the same triangle as that formed by joining $B$ to $A$ to $C$).*

2. A certain lottery is played by choosing your own set of 6 winning numbers from among the numbers 1 through 49. How many possible such combinations are there?

3. There are 12 boys and 14 girls in Ms. Brown’s class. She needs to choose 3 boys and 3 girls for a debate team. How many different teams are possible?

4. (a) In a standard deck of 52 cards, how many different 7 card hands are possible?
   (b) If you separate the cards into suits and keep only the hearts, how many different collections of 5 hearts are possible?

5. Any segment that joins two points of a circle is called a chord of that circle. For example, $PQ$ is a chord of the circle shown. Suppose six different chords are drawn in a circle. What is the greatest number of points in which the chords can intersect with each other?

6. Betty is about to order dinner at her favorite restaurant. She will order a drink, an appetizer, a main course, 2 different side items, and a dessert. If there are 10 choices for drinks, 5 appetizers, 6 main courses, 8 side items, and 5 desserts, in how many ways can Betty order her meal?

7. A school’s service club has six members. Two of them help in the Main Office each school day. What is the greatest number of school days that can pass without repeating the same pair of students?

8. Ms. Knox needs to choose 4 students to be on the Whatcom County Math Championship team. There are twenty-two students she can choose among. How many different teams can she make?
Permutations

1. How many ways are there to permute the letters in these words:
   a. DOG
   b. KITTEN
   c. MATHCLUB
   d. WCMC
   e. FAIRHAVENMIDDLESCHOOL

2. How many different six-digit numerals can be written using all of the following six digits: 4,4,5,5,5,7?

3. There are 5 seats around a table and 5 people to be seated at the table. In how many ways can they seat themselves?

4. A nursery employee wishes to plant six Golden Delicious apple trees and two Bartlett pear trees in one row. How many distinct arrangements are possible?

5. The signal mast of a ship can raise nine flags at one time (three red, two blue and four green). How many different signals can be communicated by the placement of these nine flags?

6. A teacher has made ten statements for a True-False test. Four statements are true and six are false. How many distinct answer keys could there be for the test?

7. At the Fireside Grill, you can order a meal of meat, vegetable, and a drink. If you have the following choices, how many different meals could you choose from? Meat choices: steak, chicken, or fish. Vegetable choices: potato or green beans. Drink choices: milk, lemonade, or orange juice.

8. The best possible hand in poker is a Royal Flush. It is an Ace, King, Queen, Jack, and 10 all of the same suit. What is the probability of drawing a Royal Flush out of a deck of 52 cards? (The first card can be A, K, Q, J, or 10 of any suit.)
1. At some strange math competition held in the Northwest, there are five unique team tests, a speaking test, and an individual test. If the only guideline for the order of the tests is that the individual test must come either right before or right after the speaking test, how many possible orders for the competitions are there?

2. Abby lists four consecutive multiples of some number. The average of the first two multiples is 28 and the average of the last two is 44. What is the greatest multiple on Abby’s list?

3. What is the probability of dealing a three-card poker hand where the result is one-pair (a three of a kind does not count as a one-pair)? Express your answer as a decimal to the nearest hundredth.

4. Tracy’s Trophies charges by the letter for engraving. There is one fee for each vowel and a different fee for each consonant. CAROL costs $31 to engrave. GABRIEL costs $43 to engrave. How many dollars does BRIDGET cost to engrave?

5. Staci looks at the first and fourth pages of a chapter in her book. The sum of their page numbers is 47. On what page does the chapter begin?

6. Amy, Brett and Cate each secretly write down Z, U, or T. What is the probability that Cate’s letter is different from both Amy’s letter and Brett’s letter?
Venn Diagrams

1. Out of forty students, 14 are taking English and 29 are taking Chemistry.
   a. If five students are in both classes, how many students are in neither class?
   b. How many are in either class?
   c. What is the probability that a randomly-chosen student from this group of forty students is taking only the Chemistry class?

2. In a class of 50 students, 18 take Chorus, 26 take Band, and 2 take both Chorus and Band. How many students in the class are not enrolled in either Chorus or Band?

3. There are 25 students on the Geoville Math Team. 11 play chess, 15 play tennis, while 3 play neither chess nor tennis. How many students play chess, but not tennis?

4. In a school of 320 students, 85 students are in the band, 200 students are on sports teams, and 60 students participate in both activities. How many students are involved in either band or sports?

5. Of all the mathletes at Wantagh Middle School, 80% own computers and 40% are in band. However, 10% of all the mathletes neither own computers nor are in band. What percent of all the mathletes both own computers and are in band?

6. A veterinarian surveys 26 of his patrons. He discovers that 14 have dogs, 10 have cats, and 5 have fish. Four have dogs and cats, 3 have dogs and fish, and one has a cat and fish. If no one has all three kinds of pets, how many patrons have none of these pets?

7. In Park’s School’s 8th grade, 33 students like volleyball, 34 like softball, 39 like basketball, 20 like volleyball and softball, 10 like volleyball and basketball, 8 like softball and basketball, 3 like all three sports, and 12 like none of these sports. How many students are in Park School’s 8th grade?

8. A guidance counselor is planning schedules for 30 students. Sixteen students say they want to take French, 16 want to take Spanish, and 11 want to take Latin. Five say they want to take both French and Latin, and of these, 3 wanted to take Spanish as well. Five want only Latin, and 8 want only Spanish. How many students want French only?
Important Bases to Know:
- **Base 10**: Decimal  
  \((0,1,2,3,4,5,6,7,8,9)\)  
  Example: \(3_{10}\) (in base 10 equals 3)
- **Base 16**: Hexadecimal  
  \((0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F)\)  
  Example: \(3A_{16}\) (in base 10 equals 58)
- **Base 2**: Binary  
  \((0,1)\)  
  Example: \(10_2\) (in base 10 equals 2)
- **Base 8**:  
  \((0,1,2,3,4,5,6,7)\)  
  Example: \(113_8\) (in base 10 equals 75)

1. Convert \(16_{10}\) into it’s...
   a. **binary** equivalent.
   b. **hexadecimal** equivalent.
   c. **base 8** equivalent.

2. Convert \(DE7A_{16}\) into it’s **decimal** equivalent.

3. Convert \(1001011_2\) into it’s **decimal** equivalent.

4. Convert \(2253_8\) into it’s **decimal** equivalent.

5. What is the solution to the equation \(23x - 314 = 30\), if the numbers are in base 8? **Write your answer in base 10** (decimal form).

6. Which number has the greatest **decimal** value: \(1010_2\), \(C2_{16}\), \(89_8\), or \(112_{10}\)?

7. If \(x = 101_2\), what is the **binary** value of \(y\) (the value of \(y_2\)) if \(x_{10} + y_{10} + 7_{10} = 25_{10}\)?

8. Base 5 uses the numbers 0, 1, 2, 3, and 4. Convert \(34522_5\) into it’s **decimal** equivalent.

9. Convert \(85_{10}\) into it’s **base 5** equivalent.

10. Base 29 uses the numbers and letters 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, …, Q, R, and S. Convert \(13281_{10}\) into it’s **base 29** equivalent.
Logic

1. I have a 5 scoop ice cream cone. Each of my 5 scoops is a different flavor of ice cream. The five flavors are blueberry, chocolate, strawberry, vanilla and bubble gum. You don't know what order my ice cream flavors are from top to bottom. However, here are some clues to see if you can figure out what flavors are from top to bottom:
   - The bottom flavor has 10 letters.
   - The vanilla scoop touches both the chocolate and blueberry scoop.
   - Vanilla is below the chocolate scoop but above the bubble gum scoop.

2. The Euclid City School has 1600 students. Each student takes five classes per day, and each teacher teaches four classes per day. Each class contains one teacher and 25 students. How many teachers does the Euclid City School Employ?

3. Jake has 3 more brothers than sister. How many more brothers than sisters does his sister Sara have?

4. There are six candidates (A, B, C, D, E and F) in a school election. The following facts are known to be true:
   (1) A is elected with exactly votes
   (2) B is second
   (3) F is last with 5 votes
   (4) There were a total of 100 votes cast
   (5) No two candidates received the same number of votes.

   What is the smallest number of votes B could have received?

5. Amanda arranges the digits 1, 3, 5, and 7 to write a four-digit number. The 7 is next to the 1, but not to the 5. The 3 is next to the 7, but not to the 5. The four-digit number is divisible by 5. What is Amanda’s four-digit number?

6. How many three-digit numbers satisfy all of the following?
   - The sum of the tens digit and the ones digit is 9
   - The number is even
   - The number is a multiple of 3

7. John lies on Fridays, Saturdays, and Sundays, while he tells the truth on all other days. Minnie lies on Tuesdays, Wednesdays, and Thursdays, but she is truthful on all other days. On what day of the week could they both say, “Tomorrow, I will lie”?

8. How many steps are required to break an m × n sized bar of chocolate into 1 × 1 pieces? You can break an existing piece of chocolate horizontally or vertically, but you cannot break two or more pieces at once (so no cutting through stacks).

BONUS (non-mathematical) QUESTION
You have a basket containing ten apples. You have ten friends, who each desire an apple. You give each of your friends one apple. Now all your friends have one apple each, yet there is an apple remaining in the basket. How?
1. A rectangular prism has faces with the following surface areas: 108 in$^2$, 144 in$^2$, and 192 in$^2$. What is the volume of the prism, in cubic inches?

2. Let □□m = 4m + 3. Solve for x if 10239 = □□□□□x.

3. If you choose a number that is between 20 and 80 (inclusive) at random, what is the probability you will get a number divisible by 3? Write your answer as a reduced fraction.

4. Annie has cats and birds in her house, and, for a few minutes, they all stand still enough to let her count them. If she counts 36 heads and 120 feet total, how many cats does Annie have?

5. A recipe will make 15 pancakes that are each 12 inches in diameter and 1 inch thick. If you decided to make 4 inch diameter pancakes that are 1 inch thick, how many of the smaller pancakes would this recipe make?

6. If I spell out the numbers from 0 to 99 (inclusive), what is the probability that a randomly selected number begins with a vowel? Answer as a reduced fraction.

7. Three equilateral triangles are joined to form an isosceles trapezoid. If the area of the trapezoid is 9 units, what is the perimeter of the trapezoid, to the nearest hundredth?

8. The cafeteria sells each apple at one price and each banana at another price. For 5 apples and 3 bananas Dan pays $12.75. For 3 apples and 5 bananas Chris pays $11.35. The price of one apple is how much more than the price of one banana, in cents?

9. How many whole numbers between 1 and 300 have exactly three different factors?

10. If Sam rolls two fair 6-sided dice and subtracts the smaller number from the bigger number, what is the average result of all possible rolls. Write your answer as a decimal rounded to the hundredths place.

11. A telephone call costs 33 cents for the first five minutes and 5 cents for each additional minute. If Sam pays $1.01 for two calls, for how many minutes did his two calls last when the length of each is added together?

12. Dave is three times older than Jim, and Jim is half Bill’s age. If their average age is 64, what is Jim’s age?

13. Some friends buy a video game, sharing the cost equally. If each friend pays $8, they will have $11 too much. If each friend pays $6, they will have $5 too little. What is the price of the video game in dollars?

14. Starting with 1, Sara lists the counting numbers in order but omits all those that use the digit 9. What is the 300th number on her list?

15. What is the area of the triangle formed by the x-axis, the y-axis, and the line 12x - 5y = 30?

16. The digits 1, 3, 5, 7, and 9 are each written on separate cards and placed in a bag. If three of the digits are drawn in order to make a three-digit number, what is the probability that the three-digit number is divisible by 15? Answer as a reduced fraction.
17. A, B, and C, in that order, are three consecutive whole numbers. Each is greater than 2000. A is a multiple of 4. B is a multiple of 5. C is a multiple of 6. What is the smallest possible value of A?

18. In the picture to the right, the two squares inscribed in the circle have a total area of 72 units. What is the area of the circle? Leave your answers in terms of pi.

19. What is the angle between the minute hand and the hour hand on an analog clock when it is 7:32?

20. The best possible hand in poker is a Royal Flush. It is an Ace, King, Queen, Jack, and 10 all of the same suit. What is the probability of drawing a Royal Flush out of a deck of 52 cards? (The first card can be A, K, Q, J, or 10 of any suit.)
Mock Tests
Mock Test Instructions

The following section is comprised of four Mock Whatcom County Math Championship (WCMC) tests. These tests were made by taking past WCMC tests and scaling them down to fit into a 40-minute window. Should you wish to take these tests on your own as practice make sure that you time yourself, allowing yourself only as much time for each section as I have allotted below. Doing so will allow you to be exposed to a time crunch that will be similar to that which you will experience in the actual WCMC test.

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The four unaccounted minutes that are not allotted to a specific test are meant to serve as a buffer for you – allow yourself one minute between tests to breathe and prepare for the next one.
Whatcom County Math Championship
2012 Mock Test

Individual

1. If 4 widgets and 3 thingamajigs cost $29, but 5 widgets and 2 thingamajigs cost $24, how much does one thingamajig cost?

2. How many prime numbers are there less than 100?

3. A circle of radius 5 cm sits inside a circle of radius 10 cm. What fraction of the big circle's area lies outside the small circle (write as a reduced fraction)?

4. A fair six-sided die is rolled; what is the probability that the number rolled will be a factor of 6 (write answer as a reduced fraction)?

5. If you take the middle of each side of a rhombus and connect those points, what is the best name of the type of quadrilateral that is formed?

6. \(3! = (3)(2)(1) = 6\). If we say \(3\Delta = 3! + 2! + 1!\), what is \(5\Delta\)?

7. Jane bought a puzzle book at Vowell's bookstore. It was 20% off the original price, and there was a 5% sales tax added on. She paid with a $20 bill and got back $9.92 in change. What was the original cost of the book?

8. 0 and 1 are numbers that are both perfect squares and perfect cubes; what is the next largest number that is both?

9. If an isosceles triangle has an angle of 108°, what is the measure of one of the other angles?

10. For whole numbers \(m\) and \(n\), define \(m \downarrow n = (m - n)^2\). What is \((a - b)^2 \downarrow (b - a)^2\)?

11. If \((\sqrt{16})^3 = x^2\), what is \(x\)?

12. What is the area of an isosceles triangle with sides 13, 13, and 24?
1. In the large square below, the areas marked \(a\) and \(b\) are each 4, the area of \(c\) is 16. What is the area of the trapezoid marked \(d\)?

![Diagram of a large square with areas marked a, c, and b, and a trapezoid marked d.] 

2. How many rectangles are there of all sizes in the figure below (squares count as rectangles)?

![Diagram of a grid with small squares and larger rectangles formed by combining them.]

3. The *taxicab distance* from one point to another is the shortest distance between the two points traveling only up or down or side to side along the grid. From the point A on the grid, all the points that are taxicab distance of 2 form a shape; what is the area of this shape?

![Diagram of a grid with point A marked and a grid formed by points at a taxicab distance of 2 from A.]

4. What is the angle in the degrees between the hour hand the minute hand at 8:20?
Algebra

1. Let \( a♦ = 2a + 1 \). What is \( 2♦♦♦♦♦ \)?

2. Wesley woke up at 6:32 and 24 seconds in the morning, and fell asleep at 8:12 and 16 seconds that night. How many seconds was he awake?

3. Write 58 as a binary number.

4. In the sequence 1, 8, 15, 22, \ldots\, , each number is 7 more than the number before it. In the sequence 1, 10, 19, 28, \ldots\, , each number is 9 more than the number before it. The two sequences have infinitely many numbers in common. Find the sum of the first three common numbers that are greater than 30.

Probability & Statistics

1. The average of 5 numbers is 32. If two of the numbers are removed, then the average is 30. What is the sum of the two numbers removed?

2. A jar contains 3 strawberry jellybeans, 4 orange jellybeans and 2 lemon jellybeans. Sam draws a jellybean from the jar, and then Mary draws one from those remaining. What is the probability that Mary draws a strawberry jellybean? Write the answer as a reduced fraction.

3. As the minute hand of a clock travels around the face, it passes 12 whole numbers, starting with 1. If \( n \) is the number the minute hand is on, and \( m \) is the average (arithmetic mean) of all the numbers that the minute hand has passed since it started, at what number on the clock is \( n = m + 4 \)?

4. If Brent flips 4 fair coins, what is the probability that he gets at least 3 heads? Write the answer as a reduced fraction.
Potpourri

1. Alanna is counting backwards from 10,000,000 by 9s. What is the first negative number she reaches?

2. Which is bigger, a) half of a half of a half of a half of a half, or b) half of a third of a fifth?

3. The area of triangle ABC below is 90 sq. cm. Point D is on the line segment BC, and the length of BD is 5 cm. What is the area of the triangle ADC, in square centimeters?

4. Find the least whole number $n$ greater than 30 for which:
   a) $n$ divided by 7 leaves a remainder of 1
   b) $n$ divided by 13 leaves a remainder of 3
Individual

1. If Melody rolls two six-sided dice and adds the results, what is the probability she will roll a 10 or higher? **Write your answer as a reduced fraction.**

2. What is the area of trapezoid TRAP?

3. Emma's alarm clock goes off at 5:45 AM, and she hits the snooze button. The alarm will go off again in 12 minutes, and she hits it again. After that it will go off again in 11 minutes. The alarm will continue to go off in shorter periods until, when the period between alarms is 0 minutes, at which point a fog horn will sound and Emma will have to get up. At what time does the fog horn go off?

4. If the area of the circle with Center C is 36π, what is the area of the right triangle?

5. If you count backwards from 100,000 by sevens, what is the first three digit number you will say?

6. If \( b_n \) is an arithmetic sequence, and \( b_1 = 6 \) and \( b_5 = 34 \), what is \( b_8 \)?

7. If \( n \) is divided by 7, the remainder is 5. What is the remainder of \( 2n \) divided by 7?

8. The sum of 3 consecutive odd numbers is 2013. What is the sum of all the digits of those numbers?

9. Both triangles are right. Solve for \( x \), round your answer to the nearest hundredth.

10. The least common multiple of 60 and \( n \) is 180. What is the least possible integer value of \( n \)?

11. What are the coordinates of the point where the lines \( y = 6x - 15 \) and \( y = x + 60 \) meet? **Express your answer in the form (a, b).**

12. If \( \frac{a}{b} = \frac{c}{d} \), what values for \( x \) will make \( \frac{x}{x} = 72 \)?
1. Evan has 10 tables like the one below, that fit 2 chairs along one side and 3 chairs along the other. If the tables are placed side by side, the sides that touch cannot have any chairs. Evan wants to place the tables so that they make a rectangle and each table touches another. What is the largest number of chairs Evan can have around his table?

![Diagram of tables]

2. Both figures below have the same number of squares. What is the difference in the perimeters?

![Two square grids]

3. In the circle below, BD and CE are diameters, and the measure of angle EAD is 30°. If the radius of the circle is 6, what is the length of the arc through the points C, E, and D. Leave your answer in terms of $\pi$.

![Circle with angles and diameters]

4. In the figure below, INDO is a square, and W is the center of the two semi-circles. The smaller semi-circle has a radius of half the larger semi-circle. The points on the semi-circles are evenly spaced. Find the area marked by the *. Leave your answer in terms of $\pi$.

![Square and semi-circles with points marked]
Algebra

1. In the Martian economy, 7 blorps equal a snorp, and 3 snorps equal 4 knorps. If Marvin has 24 knorps, how many blorps does he have?

2. A string of pearls is a necklace with the biggest pearl in the center, a smaller pearl on either side of the center, a smaller pearl on either side of those, and so on. Sam bought a string of pearls with 13 pearls on it, and the smallest pearls (at each end) cost $1 each, the next biggest cost $2, the next biggest cost $3, and so on. How much did Sam's necklace cost altogether?

3. In a Lucas sequence of numbers, each number is the sum of the previous two numbers in the sequence; for example: 1, 3, 4, 7, 11, 18, 29, ... What is the first missing term in this Lucas sequence: 6, ___, ___, ___, ___, 63?

4. There was an old woman who lived in a shoe with so many children and cats that she didn't know what to do. So she counted their legs (128) and she counted their heads (40), fed them all supper and sent them to bed. How many children lived in the shoe?

Probability & Statistics

1. In a race there are six racers. How many different ways can 1st and 2nd place awarded?

2. In a survey of 50 students, 22 like Harry Potter, 25 like Lord of the Rings, and 12 like neither. How many students like both Harry Potter and Lord of the Rings?

3. Instead of the normal two 6-sides dice, Matt likes to play with a fair 5-sides die and a fair 7-sided die, which he says is fair because the highest sum is still 12 and the lowest sum is still 2. What is the probability that if Matt rolls his dice and adds the results, he gets one of the three most possible outcomes? Write the answer as a reduced fraction.

4. The two dartboards below have the same size squares; in the first, the target is a circle inscribed in the square, in the second, the target is a square inscribed in a circle. If a dart hits either dartboard with equal probability, how much larger is the probability that it would hit the target on the first board? Write your answer as a decimal rounded to the hundredths place.
Potpourri

1. A basketball player made 8 baskets during a game. Each basket was worth either 2 or 3 points. How many different numbers could represent the total points scored by the player?

2. How many positive square numbers are there less than 1,000,000?

3. How many times will you use the number 8 when writing the numbers from 1 to 1000?

4. If $a$ and $b$ are positive numbers and $a^2 - b^2 = 6$, and $a - b = 2$, what is $a + b$?

Whatcom County Math Championship
2014 Mock Test

Individual

1. Jenn shoots free throws at a rate of 80%. What is the probability that she will make two free throws in a row?

2. If the diagonal of a square is $\sqrt{18}$, what is the area of the square?

3. A bag has 4 green marbles, 12 red marbles and 14 blue marbles. When a single marble is drawn at random, what is the probability that the marble is red? **Write your answer as a reduced fraction.**

4. If there are on average 12 hours of night, how many minutes are there in 1001 Arabian nights?

5. If the sum of a polygon’s interior angles is twice the sum of its exterior angles, how many sides does it have?

6. What is the value of the following: $100 - 98 + 96 - 94 + \ldots + 8 - 6 + 4 - 2$?

7. How many prime numbers are there between 100 and 150?

8. 36% of what number is 27?

9. What is the area of an isosceles triangle with sides 13, 13 and 24?

10. The sum of two numbers is 58, and the difference between them is 12. What is the larger of the two numbers?

11. In a single-elimination tournament with 32 teams, the winning team will have won how many games?

12. How many different perfect square numbers and perfect cube numbers are there between 1 and 1001?
Algebra

1. How many circles will there be at step 20?

2. When a positive two-digit number has its digits reversed to form a new two-digit number, the sum of the two is 154. What is the largest possible value of the original number?

3. The digital root of a number is found by adding all of the digits of a number, then repeating the process until you get a single digit number (for example, the digital root of 3851 is $3 + 8 + 5 + 1 = 17, 1 + 7 = 8$). What is the digital root of 92014?

4. The line $m$ goes through the points $(4, 21)$ and $(28, 3)$, while the line $n$ goes through the point $(16, 27)$ and has a slope of 3. Lines $m$ and $n$ intersect at the point $P$. Give the coordinates $(x, y)$ for $P$.

Geometry

1. A rectangular prism has faces with the following surface areas: 108 in², 144 in² and 192 in². What is the volume of the prism, in cubic inches?

2. In the picture below, the shaded circle has an area of $81\pi$ cm², and the larger circle has a diameter that is 6 centimeters longer than the diameter of the shaded circle. What is the unshaded area of the larger circle, in terms of $\pi$?

3. What is the angle of between the minute hand and the hour hand on an analog clock when it is 10:55?

4. A square of area 100 is cut into four congruent triangles and a smaller square. If the side lengths of the legs of one of the triangles are in a ratio of 2:1, what is the area of the smaller square?
Probability & Statistics

1. If Amelia rolls two fair 6-sided dice and multiplies the results, what is the probability that they answer will be evenly divisible by 4? Write your answer as a reduced fraction.

2. Tamara is walking through the city from Start to Finish. At each intersection, she flips a coin to decide which way to go, south or east (if she has no choice, she keeps going). What is the probability that she passes through the point x on her way from S to F?

3. The Fibonacci numbers are those that follow the pattern 1, 1, 2, 3, 5, 8…. If you choose a number from 1 to 10000 at random, what is the probability that you will choose a Fibonacci number? Write your answer as a reduced fraction.

4. If you choose a number that is from 20 through 80 at random, what is the probability you will get a number divisible by 3? Write your answer as a reduced fraction.

Potpourri

1. Ben is counting backwards from 1,000,000 by thirteens. What is the first negative number he reaches?

2. All of the triangles in the figure below are right triangles, and all of the circles have a radius of 1. How long is x? Give your answer to the nearest integer.

3. How many different ways can you arrange the letters in the word ARRANGE?

4. What is the solution to the equation $23x - 314 = 30$, if the numbers are in base 8? Write your answer in base 10.
1. Noah buys 3 labyrinth books at Vowell’s Bookstore that cost $3.95 each, but because Vowell’s is having an amazing sale, the books are 60% off. If Noah gives the clerk a ten-dollar bill, how much change does he get back?

2. Drake’s Ice Cream sells chocolate, vanilla, blueberry, cinnamon and butterscotch ice cream. If Alexia gets a double scoop dish with different flavors and it does not matter which scoop is on top, how many different possible combinations could she get?

3. If the angles of a triangle are in a 1:2:2 ratio, what is the measure of the bigger angle?

4. How many vertices are there on a four-dimensional hypercube?

5. What is the square of the cube root of the reciprocal of 3.375? Express your answer as a reduced fraction.

6. The inverse of 3 mod 5 is 2, because 2*3 mod 5 is 1. What is the inverse of 3 mod 13?

7. What is 25% of \(\frac{3}{4}\)? Write your answer as a reduced fraction.

8. If the sequence 8, m, 72… is a geometric sequence and m is positive, what is the next term in this arithmetic sequence 8, m, ____?

9. Four identical squares and one rectangle are placed together to form one large square as shown below. If the area of one of the small squares is 36, what is the area of the rectangle?

10. The parabola given by the equation \(y = x^2 - 6x + m\) will have no x-intercepts if m is greater than n. What is the smallest value of n?

11. Rapunzel’s hair is 2 feet long on Saturday March 14th, and grows a specific amount every day. By March 30th, her hair has tripled in length. How many inches does it grow every day?

12. What is the sum of the x-intercepts and y-intercept of the parabola given by \(y = x^2 - 10x - 24\)?
Algebra

1. What is the difference between the sum of the first 2015 even counting numbers and the sum of the first 2015 odd counting numbers?

2. If 4, ___, ___, 72 is a geometric sequence, what is the third term? **Round to the nearest hundredth.**

3. $8^x = 27$. What is $4^x$?

4. The 9th term of the geometric sequence 24, 36, 54, … will be a number in the form $\frac{3^n}{2^m}$. What is $-m$?

Geometry

1. If a circle has a center at the point (6,17) and is tangent to another congruent circle at the point (8,14), what are the coordinates of the center of the second circle?

2. A square is surrounded by equilateral triangles. What is the measure of $x$, angle ABC?

3. What is the area between the x-axis, the y-axis, and the function $y = 4 - |x - 2|$?

4. If the four coordinates of a rectangle are (4,6), (8,4), (-2,-6) and (m,n), what is $m + n$?
Probability & Statistics

1. What is the probability that a randomly drawn positive factor of 72 is less than 5? Write your answer as a reduced fraction.

2. Using the letters A, B, E, I, L, and N, we can form 720 six-letter “words”. If these “words” are arranged in alphabetical order, the “word” BLAINE occupies what position?

3. A rectangle is drawn connecting the points (0,0), (6,0), (6,2) and (0,2). If a point (x,y) is chosen at random in this rectangle, what is the probability that x < y ? Write your answer as a reduced fraction?

4. Robin was throwing darts at the regular octagon target shown below. If she is throwing her darts at random, what is the probability that she will hit the shaded region? Round your answer to the nearest hundredth.

![Octagon Target Image]

Potpourri

1. A digital clock shows 1:35. This is the first time after midnight when all three digits are different odd numbers. How many time between midnight and 10:00 AM will the digits be three different odd numbers?

2. If the pattern below continues, how many stars will there be in figure 10?

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3. What is the sum of the digits of the square of 111,111,111?

4. Each morning before school, Grace bought either a 50-cent muffin or a 75-cent bagel. Her total cost for the five-day week was a whole number of dollars. How many bagels did she buy?
Mock Test
Answer Sheets
## Mock WCMC Test Answers

Name: ____________________________________________________________
Grade: _________  Mock Test Year: ________________________________

### Individual Test

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### Mental Math

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# Mock WCMC Test Answers

Name: __________________________________________________________

Grade: _________  Mock Test Year: ________________________________

## Individual Test

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## Mental Math

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**Prob. & Stat.**

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**Geometry**

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**Potpourri**

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Mock WCMC Test Answers

Name: _______________________________________________
Grade: _________  Mock Test Year: ______________________

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Designed by Courtney Knox
For additional copies please contact her directly.
E:mail: knoxc3@students.wwu.edu
Phone: (360) 628-3131

Disclaimer: Courtney did not create the math problems in this workbook. The math problems came from the MOEMS Contest Problems book, Volume 3, and old Whatcom County Math Championship tests.
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Area and Perimeter Answers

1. 22.28 inches
2. $N = 12$
3. 25 square centimeters
4. 48 centimeters
5. 75.36 feet
6. 48 feet
7. $\sqrt{8}$ inches OR $2\sqrt{2}$ inches if you simplify it
8. 2 gallons
9. 28.5 square feet
10. 160 pancakes

Work shown on the following pages
Answers have been circled
1. \( b, c, d, e = \text{arc length of} \quad \frac{1}{4} \text{ of a circle with radius 1} \)

\[ b + c + d + e = \text{circumference of circle with radius 1} \]

\[ 2\pi = 6.28 \]

\[ 6.28 + (4 \cdot 4) = 22.28 \]

Center of circle travels 22.28 inches.

2. \( \frac{y}{w} = 3 \)

\( x = 10, y = 7 \)

\( z = 6, w = 2 \)

Factors of 35:

\((1, 35), (5, 7)\)

3. \( 100 = 10 \times 10 \)

\( 5 \times 5 = 25 \text{ sq. cm} \)
4. \[ \text{area } ABCD = 144 \]
\[ \text{(area I) } \cdot 3 = \text{area I} \]
\[ \text{area II} + \text{area I} = 144 \]
\[ \text{(area I) } \cdot 3 + \text{area I} = 144 \]
\[ \text{area I is a square, } \frac{EA}{AD} = \frac{BC}{DC} = 2 \Rightarrow \text{area I} = 36 \]
\[ \text{(area I) } \cdot 3 = \text{area II} \]
\[ (36) \cdot 3 = 108 = BC \times BE, \text{ area I} = 36 \]
\[ \frac{108}{6} = 18 \]
\[ \text{Perimeter of II: } 2(6 + 18) = 48 \text{ cm} \]

5. \[ \text{Circumference}_1 = \frac{1000}{\pi} \text{ ft} \]
\[ = \frac{2 \pi r}{\pi} = 2r \text{ ft} \]
\[ \text{diameter}_1 (d) = \frac{1000}{\pi} \text{ ft} \]
\[ \text{running 3 ft outside the edge of the track adds 6 ft to the diameter} \]
\[ \text{diameter}_2 (d) = \frac{1000}{\pi} + 6 \text{ ft} \]
\[ \text{Circumference}_2 = \frac{1000}{\pi} + 6 \pi \text{ ft} \]
\[ \text{Circumference}_2 - \text{Circumference}_1 = \frac{6 \pi}{\pi} - 1000 \]
\[ = 6 \pi \times 4 \text{ laps} \]
\[ = 24 \pi \text{ ft} \]
\[ \approx 75.36 \text{ ft} \]
10

\[ a^2 + b^2 = c^2 \]
\[ (6)^2 + (8)^2 = c^2 \]
\[ 36 + 64 = 100 = c^2 \]
\[ 10 = c \]

7

Area of sides

8

Area of room: \( \pi r^2 = \pi (10)^2 = 100\pi \)

Area of unpainted region: \( \pi r^2 = \pi (8)^2 = 64\pi \)

1 gal. of paint = 100\pi sq. ft. of painted floor space

\[ \frac{64\pi}{36\pi} = 1.778 \]

Need 2 more gallons of paint
9. \[ x^2 + x^2 = (10)^2 \]
\[ 2x^2 = 100 \]
\[ x^2 = 50 \]
\[ x = \sqrt{50} \]

Area of square with side lengths of \( \sqrt{50} \) ft = 50 sq ft

Area of circle with diameter \( 10 \) ft:
\[ A = \pi \left( \frac{10}{2} \right)^2 \]
\[ A = \pi \left( 5 \right)^2 \]
\[ A = 25\pi \]
\[ 25\pi - 50 \approx 78.5 - 50 = 28.5 \text{ sq ft} \]

10. Pancakes:

Area of pancakes with diameter 8 in:
\[ A = \pi \left( \frac{8}{2} \right)^2 \]
\[ A = \pi (4)^2 \]
\[ A = 16\pi \]

10 \( \cdot 16\pi = 160\pi \text{ ft}^2 \)

Batter makes 160\( \pi \) sq in of pancakes

Not drawn to scale

Area of pancakes with diameter of 2 in:
\[ A = \pi \left( \frac{2}{2} \right)^2 \]
\[ A = \pi (1)^2 \]
\[ A = \pi \]

Batter: \( 160\pi \text{ ft}^2 = 160 \text{ pancakes} \)

Area of pancake: \( \pi \text{ ft}^2 = 100 \text{ pancakes} \)
Volume Answers

1. 1728 cubic inches
2. 3153600000 cubic yards per year
3. 64 times
4. 1728 cubic inches
5. 5451776000 cubic yards
6. 48385 cubic inches
7. 0.04 cm.
8. 47.1 cubic inches

Work shown on the following pages
Answers have been circled
1. cubic foot = 1 foot wide \times 1 \text{ foot long} \times 1 \text{ foot high}
   = 12 \text{ inches wide} \times 12 \text{ inches long} \times 12 \text{ inches high}
   = (12 \times 12 \times 12) \text{ inches}^3
   = 1728 \text{ cubic inches}

2. cubic yard = 1 \text{ yard wide} \times 1 \text{ yard long} \times 1 \text{ yard high}
   = 3 \text{ feet wide} \times 3 \text{ feet long} \times 3 \text{ feet high}
   = (3 \times 3 \times 3) \text{ feet}^3
   = 27 \text{ cubic feet}

   \[
   \frac{2700 \text{ cubic feet}}{1 \text{ cubic yard}} = \frac{100 \text{ cubic yards}}{27 \text{ cubic feet}}
   \]

3. volume of earth = \frac{4}{3} \pi (4000)^3
   volume of moon = \frac{4}{3} \pi (1000)^3

   \[
   \frac{\frac{4}{3} \pi (4000)^3}{\frac{4}{3} \pi (1000)^3} = \frac{(4)^3}{(1)^3} = 64 = 64 \text{ times}
   \]
Factors of 108: (2, 54), (3, 36), (4, 27), (6, 18), (9, 12)

Possible y-values (numbers that go into 144 evenly) are circled.

Using the possible y-values, find possible z-values.

Plug the possible z-values in with the corresponding x-values. ("Corresponding" refers to the pairs of factors of 108, i.e., if y = 6 then the corresponding x-value is 18) to see which works.

\[
\begin{align*}
y = 9 & \quad \text{corresponding } x = 12 \\
2y = 144 & \quad \Rightarrow \quad z = 16 \quad \text{same, } y = 9 \text{ works!} \\
2x = 192 & \quad \Rightarrow \quad \frac{192}{2} = x = \frac{192}{16} = 12 = x
\end{align*}
\]

\[
x = 12 \\
y = 9 \\
z = 16
\]

\[
\text{Volume } = (12)(9)(16) = 1728 \text{ in}^3
\]

5 cubic mile = 1 mile wide x 1 mile long x 1 mile high

= 1760 yards wide x 1760 yards long x 1760 yards high

= \((176)^3\) yards

= \(6,451,776,000\) cubic yards

6 1 inch cube = 1 inch wide x 1 inch long x 1 inch high

= 1 cubic inches

1 foot cube = 12 inches wide x 12 inches long x 12 inches high

= 1728 cubic inches

1 yard cube = 36 inches wide x 36 inches long x 36 inches high

= 46,656 cubic inches

\(1 + 1728 + 46,656 = 48,385\) cubic inches
1 cm cube = 1 cm wide x 1 cm long x 1 cm high
= 1 cubic cm

Let y equal the thickness of the sheet
cube = sheet = 1.25 cm wide x 20 cm long x y cm high
1 cubic cm = (1.25 x 20 x y) cm³

\[ \frac{1}{25} = 0.04 = y \rightarrow 0.04\text{ cm} \]

Stack of 40 washers

\[ = \left( \frac{1}{8} \text{ in} \right) 40 = 5 \text{ in} = h \]

Volume = \( \pi r^2 h \)

\[ = \left( \text{washer surface area} \right) h \]

Washer surface area = \( \pi (2)^2 - \pi (1)^2 \)
\[ = 4 \pi - \pi \]
\[ = 3 \pi \]

Volume = \( \left( \text{washer surface area} \right) h \)

\[ = \left( 3 \pi \right) 5 \]
\[ = 15\pi \text{ in}^3 \approx 47.1 \text{ in}^3 \]
Decimals, Fractions, and Percents Answers

1. 
   a. \(\frac{20}{19}, 1.108, 6.8, 1\)
   b. \(\frac{21}{16}, 1.723, 21.42, 15\)
   c. \(\frac{18}{5}, 12.96, 128.52, 63\)
   d. \(\frac{3}{4}, 0.563, 73.44, 216\)
2. $24.60
3. 30
4. 3
5. \(\frac{57}{88}\)
6. 1.25 miles
7. \(\frac{14}{19}\)
8. 129%
9. 35
10. 60

Work shown on the following pages
Answers have been circled
1.

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<tr>
<th>Part</th>
<th>$\frac{18}{19}$ in reduced fraction form</th>
<th>$\frac{18}{19}$ in decimal form</th>
<th>$\frac{18}{19}$ x fraction equivalent correct</th>
<th>Remainder $(\text{division})$</th>
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<tr>
<td>q</td>
<td>$\frac{20}{19}$</td>
<td>$(20)^2 \div (19)^2 \approx 1.108$</td>
<td>$20 \times (0.34) \approx 6.8$</td>
<td>$19 \div 20 ; R = 1$</td>
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<tr>
<td>b</td>
<td>$\frac{63}{48} = \frac{21}{16}$</td>
<td>$(63)^2 \div (48)^2 \approx 1.723$</td>
<td>$63 \times (0.34) \approx 21.42$</td>
<td>$48 \div 63 ; R = 15$</td>
</tr>
<tr>
<td>c</td>
<td>$\frac{378}{105} = \frac{18}{5}$</td>
<td>$(378)^2 \div (105)^2 \approx 1.296$</td>
<td>$378 \times (0.34) \approx 128.52$</td>
<td>$105 \div 378 ; R = 63$</td>
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<tr>
<td>d</td>
<td>$\frac{216}{288} = \frac{3}{4}$</td>
<td>$(216)^2 \div (288)^2 \approx 0.566$</td>
<td>$216 \times (0.34) \approx 73.44$</td>
<td>$288 \div 216 ; R = 216$</td>
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2.

18.95 mi/gal

\[ \frac{379}{18.95} \approx 20 \]

\[ 20 \times (1.23) = \$24.60 \]

3.

\[ \frac{1}{5} = \frac{1}{6} + \frac{1}{N} \]

\[ \frac{1}{5} - \frac{1}{6} = \frac{1}{N} \]

\[ \frac{6}{30} - \frac{5}{30} = \frac{1}{N} \]

\[ \frac{1}{30} = \frac{1}{N} \]

\[ 30 = N \]

4.

Jess won 20% \rightarrow Emily won 80%

Emily won 12 matches \rightarrow 12 matches = 80%

Let \( x \) = total # of matches

\[ 12 = x \times (0.8) \]

\[ \frac{12}{0.8} = x \rightarrow x = 15 \]

15 total matches

Emily won 12.

Jess won \((15 - 12) = 3\)
5. \[
\frac{3}{4 + \frac{3}{4}} = \frac{3}{\frac{19}{4}} = \frac{12}{19}
\]
\[
\frac{12}{19} = \frac{88}{19}
\]

6. 8 days
82.5 mi
8.25 hr/day
\[
\frac{8.25 \text{ hr}}{1 \text{ day}} \cdot 8 \text{ days} = 66 \text{ hrs.}
\]
8.25 mi
66 hr
\[
\frac{8.25 \text{ mi}}{66 \text{ hr}} = 0.125 \text{ mi/hr}
\]

7. \[
\left(\frac{7}{19} \cdot \frac{13}{44}\right) + \left(\frac{7}{19} \cdot \frac{19}{44}\right) + \left(\frac{7}{19} \cdot \frac{25}{44}\right) + \left(\frac{7}{19} \cdot \frac{31}{44}\right)
\]
\[
\frac{91}{836} + \frac{133}{836} + \frac{175}{836} + \frac{217}{836} = \frac{616}{836} = \frac{14}{19}
\]

8. 5" x 7" \rightarrow \text{area} = 35 \text{ sq in}
8" x 10" \rightarrow \text{area} = 80 \text{ sq in}
80 - 35 = 45
\[
\frac{45}{35} = \frac{9}{7} = 1.29 = 129\%
\]

9. Let \( x \) = total # of apples picked and \( y \) = total # of apples picked by Penny
Jenny \( \rightarrow \frac{x}{4} \) \( \rightarrow \frac{x}{3} - 7 = \frac{x}{4} \) \( \rightarrow \frac{4x}{3} - 28 = x \) \( \rightarrow 4x - 84 = 3x \) \( \rightarrow x = 84 \)
Lenny \( \rightarrow \frac{x}{5} \) \( \rightarrow \frac{x}{5} + \frac{x}{4} = y \) \( \rightarrow x = 84 \) \( \rightarrow \frac{84}{3} + \frac{84}{4} = y \) \( \rightarrow 28 + 21 + y = 84 \) \( \rightarrow y = 35 \)

10. Let \( n \) = some whole number
\[
76(0.95) = 57 = n(0.95) \rightarrow \frac{57}{0.95} = n \rightarrow n = 60
\]
Mean, Median, Mode, and Range Answers

1.
   a. 15.429, 15, no mode, 7
   b. 1.016, 1.0001, 1, 0.1
   c. 3.444..., 4, \{0, 2, 4, 6\}, 7
   d. 3.117, 3.125, 3, 5.667
2. 83.84
3. 51
4. 38
5. 54
6. 85
7. The $84.99 one
8. 36
9. A = 1.8
10. 4

Work shown on the following pages
Answers have been circled
1. Part a: 12, 13, 14, 15, 17, 18, 19
   mean: \[ \frac{12+13+14+15+17+18+19}{7} \approx 15.429 \]
   median: 15
   mode: no mode (every # only occurs once)
   range: 19 - 12 = 7

Part b: 1.1, 1, 1.01, 1, 1.001, 1, 1.0001
   mean: \[ \frac{1.1+1+1.01+1+1.001+1+1.0001}{7} \approx 1.016 \]
   median: 1.0001
   mode: 1 (occurs 3 times)
   range: 1.1 - 1 = 0.1

Part c: 0, 0, 2, 2, 4, 4, 6, 6, 7
   mean: \[ \frac{0+0+2+2+4+4+6+6+7}{9} \approx 3.444 \]
   median: 4
   mode: 0, 2, 4, 6 (all occur 2 times)
   range: 7 - 0 = 7

Part d: \[ \pi, \sqrt{\pi}, 3.125, 3, 3.22, 3.3, 3.1 \]
   mean: \[ \frac{\pi+3+3.125+3+3.22+\frac{\pi}{3}+3.3}{7} \approx 3.117 \]
   median: 3.22
   mode: 3 (occurs 2 times)
   range: 7 - \frac{1}{3} = \frac{12}{3} \approx 5.667

\( a \) \[ \frac{85.5 + 65.75 + 92.5 + 91.6}{4} \approx 83.84 \]

\( \frac{b}{3} \) \[ \frac{3+6+9+\ldots+93+96+99}{33} = 51 \]

Shortcut:
equivalent to taking the average of the greatest and least common multiple: \( \frac{99 + 3}{2} = 51 \)
4. Let \( x \) be the number of hits that Chad has on the final day of the week.
\[
\frac{37 + 29 + 37 + 56 + 45 + 38 + x}{7} = 40
\]
\[
242 + x = 280
\]
\[
x = 38
\]

5. (12 + 16 + 20 + \ldots + 88 + 92 + 96)/22 = \frac{54}{22}
shortcut: equivalent to taking the average of the greatest and least common 2-digit multiple: \((96 + 12)/2 = 54\)

6. Let \( x \) be the class average.
Amanda’s score: \( x - 5 \)
Barb’s score: \( x + 8 \)
Colin’s score: \( 82 \)
\[
\frac{(x-5) + (x+8) + 82}{3} = x
\]
\[
x - 5 + x + 8 + 82 = 3x
\]
\[
2x + 85 = 3x
\]
\[\boxed{85} = x\]


8. Let \( x \) be the missing number.
\[
\frac{19 + 21 + 24 + 28 + x}{5} = 25
\]
\[
89 + x = 125
\]
\[
x = 36
\]

9. Since we know there is a mode, we know \( A \) must be one of the given numbers (1.8, 1.6, 2.1, 1.7). Knowing that, we know \( A \) is the mode. Now test all possibilities:
\[
\boxed{A = 1.8} \quad \text{mean} = 1.8, \quad \text{median} = 1.8, \quad \text{mode} = 1.8
\]

10. Let \( x \) be the number of runs in their final game.
\[
\frac{6 + 2 + 5 + 7 + 11 + 4 + 5 + 8 + 6 + 7 + 5 + x}{12} = 6
\]
\[
68 + x = 72
\]
\[
x = 4
\]
Primes, Squares, and Cubes Answers

1. 1999
2. 20
3. 19
4. 144
5. 196
6. 3
7. 5
8. 86
9. 4, 9, 25, 49
10. 112

Work shown on the following pages
Answers have been circled
1. \( x + y = 2001 \)
   
   either \( x \) or \( y \) must be even because 2001 is odd
   
   \( 2 \) is the only even prime
   
   \( 2 + y = 2001 \)
   
   \( y = 1999 \)
   
   1999 is the larger of the two primes

2. 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

   0, 0, 2, 0, 0, 2, 0, 0, 2, 0, 0, ...

   \( \rightarrow \) every \( \frac{1}{8} \) # is even
   
   \( \rightarrow \frac{1}{8} \) of the #’s are even
   
   \( \rightarrow \frac{5}{8} \) of the #’s are odd

   \( \frac{30 \times 2}{3} = \frac{60}{3} = 20 \) odd numbers

3. \( 5 \times P + 7 \times Q = 109 \)

   \( 5P + 7Q = 109 \)

   either \( 5P \) or \( 7Q \) must be even because 109 is odd

   Case 1: \( 5P \) is even
   
   \( \rightarrow P \) is even
   
   \( \rightarrow P = 2 \) (only even prime)
   
   \( \rightarrow 5(2) + 7Q = 109 \)
   
   \( \rightarrow 7Q = 99 \) IMPOSSIBLE

   Case 2: \( 7Q \) is even
   
   \( \rightarrow Q \) is even, \( Q = 2 \) (only even prime)
   
   \( \rightarrow 5P + 7(2) = 109 \)
   
   \( \rightarrow 5P = 109, \ P = 19 \)
4. 

\[ \begin{array}{|c|c|c|} 
\hline 
\text{Row} & \# \Delta_1 & \text{Total } \# \Delta_1 \\ 
\hline 
1 & 1 & 1 \\ 
2 & 3 & 4 \\ 
3 & 5 & 9 \\ 
4 & 7 & 16 \\ 
5 & 9 & 25 \\ 
\hline 
\end{array} \]

\[ (\text{row})^2 = \text{Total } \# \Delta_1 \]

5. 

\[
\begin{align*}
100 & \leq x \leq 200 \\
100 & = 10^2 \\
x & = y^2 \\
11^2 & = 121 \\
x & = 121 \\
12^2 & = 144 \\
13^2 & = 169 \\
14^2 & = 196 \\
15^2 & = 225 \\
\text{only square } & \text{ between 100 and 200 divisible by 7} \\
7 \times 28 & = 196
\end{align*}
\]

6. 

<table>
<thead>
<tr>
<th>Prime $x$</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>11</th>
<th>13</th>
<th>17</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 - $x$</td>
<td>38</td>
<td>37</td>
<td>35</td>
<td>33</td>
<td>29</td>
<td>27</td>
<td>23</td>
<td>21</td>
</tr>
<tr>
<td>Is 40 - $x$ prime?</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>N</td>
</tr>
</tbody>
</table>

3 ways

7. Odd # of factors $\rightarrow$ our # is a perfect square.

(For example, factors of 16: 1, 2, 4, 8, 16, $4^2 = 16$)

Let $N =$ our whole number(s)

Factors of $N$: 1, $x$, $N$ where $N$ is a perfect square and $x$ is its square root

$\rightarrow$ Since there are no other factors, we know that $x$ is prime.
7. Continued...

N is a perfect square
\[ x = \sqrt{N} \] and is prime

There are 5 whole numbers between 1 and 150 that have exactly 3 factors.

\[ \begin{array}{c|c}
\text{possible } x \text{'s} & \text{corresponding } N \text{'s} \\
\hline
2 & 4 \\
3 & 9 \\
5 & 25 \\
7 & 49 \\
11 & 121 \\
13 & 169 \rightarrow \text{too large}
\end{array} \]

8. \( x \) : 1, 2, 3, ..., 74, 76

Perfect squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, ...

Perfect cubes: 1, 8, 27, 64, ...

→ perfect squares and perfect cubes must be "deleted"

→ \( x \) must always have 75 numbers in it

Deleting #'s:
\[ x - 1 \rightarrow x : 2, 3, ..., 74, 75, 76 \]
\[ x - 4 \rightarrow x : 2, 3, 5, ..., 74, 75, 76, 77 \]

\[ \vdots \]
\[ x - 64 \rightarrow x : ..., 60, 62, ..., 81, 82, 83, 84, 86 \]
\[ x - 81 \rightarrow x : ..., 82, 83, 84, 85, 86 \]

9. See answer for #7.

Qualifying \( N \)'s: 4, 9, 25, 49, 121

\[ \text{Less than 50} \]

Counting numbers less than 50 that have exactly three factors:

\[ 4, 9, 25, 49 \]

10. Primes between 50 and 60:

53, 59

\[ 53 + 59 = 112 \]
Probability Answers

1. 1/128 (or approximately 0.007813)
2. 500/3 (or approximately 166.67)
3. 7/26 (or approximately 0.269)
4. No
5. 25/31 (or approximately 0.806)
6. 97/177 (or approximately 0.548)
7. 19/40 (or approximately 0.475)
8. 1/1906884 (or approximately 0.00000052)
9. 781/1024 (or approximately 0.763)
10. 31/32 (or approximately 0.969)
11. 1/425 (or approximately 0.0024)
12. 1/5525 (or approximately 0.000181)

Work shown on the following pages
Answers have been circled
1. Let \( P(x) \) equal the probability that you will get a heads on flip \# \( x \).

\[
P(1) = \frac{1}{2}, \quad P(2) = \frac{1}{2}, \quad P(3) = \frac{1}{2}, \quad P(4) = \frac{1}{2}, \quad P(5) = \frac{1}{2}, \quad P(6) = \frac{1}{2}, \quad P(7) = \frac{1}{2}
\]

\[
P(1) \times P(2) \times P(3) \times P(4) \times P(5) \times P(6) \times P(7) = \text{probability you will get 7 heads in a row}.
\]

\[
\left(\frac{1}{2}\right)^7 = \frac{1}{128} \approx 0.007813
\]

2. Possible outcomes for sums of a roll of two dice:

<table>
<thead>
<tr>
<th>Dice 1</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>

The sums that equal 7 have been circled. Six of the 36 possible sums equal 7.

Probability of rolling two dice and getting a sum that equals 7 = \( \frac{6}{36} = \frac{1}{6} \).

1000 rolls \( \times \frac{1}{6} \) probability = \( \frac{1000}{6} = \frac{500}{3} \approx 166.67 \)

3. # of diamond cards in a deck = 13
# of aces & spades = 1

Probability of picking a diamond or the ace & spades = \( \frac{13+1}{52} \times \frac{1}{52} = \frac{14}{52} \times \frac{1}{52} \approx 0.0097769 \)
4. No

From problems #1 and #2, we can tell that the probability of getting three heads is:

\[
P(H) \times P(H) \times P(H) = \frac{1}{8}
\]

And the probability of rolling two dice and getting a sum of 7 is:

\[
\frac{6}{36} = \frac{1}{6}
\]

\[
\frac{1}{8} \neq \frac{1}{6}
\]

5. $10 in quarters = \binom{10}{4} = 40 quarters$
$10 in nickels = \binom{10}{20} = 200 nickels$
$10 in pennies = \binom{10}{100} = 1000 pennies$

1240 total coins

Probability of picking a penny = \[
\frac{1000}{1240} = \frac{25}{31} \approx 0.806
\]

6. There are 177 total pages in the book. 97 of them contain the digit 1:

1, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 31, 41, 51,
61, 71, 81, 91, 100, 101, 102, 103, 104, 175, 176, 177

So the probability of choosing one of those pages is:

\[
\frac{97}{177} \approx 0.548
\]

7. #s between 11 and 50 that are divisible by 3:

12, 18, 21, 24, 27, 33, 36, 39, 42, 45, 48

#s between 11 and 50 that are divisible by 5:

20, 25, 30, 35, 40, 45, 50

# between 11 and 50 that are divisible by both 3 and 5:

15, 30
So, since there are 11 #’s between 11 and 50 that are divisible by 3, 6 #’s between 11 and 50 that are divisible by 5, and 2 #’s between 11 and 50 that are divisible by both 3 and 5, the probability & choosing one of them is: 
\[
\frac{11+6+2}{40} = \frac{19}{40} \approx 0.475
\]

8 Player picks 5 numbers.

When the first winning number is chosen, he has a \( \frac{2}{49} \) chance & matching it. If he matches it, he has 4 numbers left to match out of the 48 possible winning numbers left. When the second winning number is chosen, he has a \( \frac{4}{48} \) chance & matching it. If he matches it, he has 3 numbers left to match out of the 47 possible winning numbers left. And so on and so forth.

Let \( P(x) \) represent the probability of matching the \( x^\text{th} \) winning number.

\[
P(1) \times P(2) \times P(3) \times P(4) \times P(5) = \text{probability of matching all 5 winning numbers}
\]

\[
\frac{2}{49} \times \frac{4}{48} \times \frac{3}{47} \times \frac{2}{46} \times \frac{1}{45}
\]

\[
\frac{120}{2,288,260,800} = \frac{1}{190,688,4} \approx 0.000000052
\]

9 Probability of not choosing a spade from the first deck = \( \frac{3}{4} \)

Probability of not choosing a spade from any deck =

\[
\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{243}{1024}
\]

Therefore, the probability of choosing a spade = \( \frac{1024-243}{1024} \approx 0.763 \)
10. Only 2 cases where the couple will not have their wish:
1. They have all girls
2. They have all boys

Probability of having all girls or all boys is similar to finding the probability of getting all heads or all tails after flipping a coin six times. Probability of having all girls or all boys = \( \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{64} \)

So, the probability for not having one of those two cases is:
\[
\frac{64}{64} - \frac{1}{64} - \frac{1}{64} = \frac{62}{64} = \frac{31}{32} \approx 0.969
\]

11. First pick determines the card you want three-of-a-kind 8. After that, there are 3 matching cards in the 51 remaining. And, assuming you draw one of those 3 matching cards for your second card, after that there are only 2 matching cards out of the remaining 50.

\[
\frac{52}{52} \times \frac{3}{51} \times \frac{2}{50} = \frac{512}{152600} = \frac{1}{425} \approx 0.0024
\]

12. The probability of drawing an ace on your first draw is \( \frac{4}{52} \). After that, there are 3 other aces left in the 51 remaining. And, assuming you draw one of those 3 aces for your second card, after that there are only 2 other aces left in the 50 remaining cards.

\[
\frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} = \frac{24}{132600} = \frac{1}{5525} \approx 0.000181
\]
Algebra Answers

1. 12
2. 58
3. 21 nickels
4. 10
5. $79
6. 30
7. 9
8. $35
9. 9 deer
10. 50 cents
11. 21 years old
12. 8 minutes
13. 12 marbles
14. 3.4
15. 18 boys

Work shown on the following pages
Answers have been circled
1. Let \( n \) be some number.

\[
8n = n + 84 \\
7n = 84 \\
n = 12
\]

2. \( 3.0 \div 26 = 8 \) \( \Rightarrow \) \( 3.0 + 26 = 33 \)

\[
30 \div 11 = 33 \\
33 + 25 = 58
\]

3. Let \( n \) be the number of dimes.

Number of nickels = \( 3n \)

\[
\begin{align*}
\text{Value of dimes} &= 10n \\
\text{Value of nickels} &= 15n
\end{align*}
\]

\[
\begin{align*}
25n &= 175 \\
3n &= 21
\end{align*}
\]

4. Let \( n \) be Carol's number.

\[
\begin{align*}
(((n \times 4) + 8) \times \frac{1}{4}) - 8 &= 4 \\
(((n \times 4) + 8) \times \frac{1}{4}) &= 12 \\
(n \times 4) + 8 &= 48 \\
(n \times 4) &= 40 \\
n &= 10
\end{align*}
\]
5. Daniel has \( n \) dollars
Luke has \( n + 48 \) dollars
Rachel has \( (n + 48) - 21 \) dollars

\[
\text{Daniel} + \text{Luke} + \text{Rachel} = 168
\]

\[
n + (n+48) + ((n+48) - 21) = 168
\]

\[
n + n + 48 + n + 48 - 21 = 168
\]

\[
(n + n + n) + (48 + 48 - 21) = 168
\]

\[
3n + 75 = 168
\]

\[
3n = 93
\]

\[
n = 31 \quad \rightarrow \quad \text{Luke} = n + 48 = 31 + 48 = 79
\]

6. Let the median of the 7 consecutive integers be \( x \)

\[
(x-3) + (x-2) + (x-1) + (x) + (x+1) + (x+2) + (x+3) = 105
\]

\[
x - 3 + x - 2 + x - 1 + x + x + 1 + x + 2 + x + 3 = 105
\]

\[
(x + x + x + x + x + x + x) + (-3 - 2 - 1 + 1 + 2 + 3) = 105
\]

\[
7x + 0 = 105
\]

\[
x = 15
\]

Least: \((x-3) = 12\) \quad \left\{\begin{array}{c} 18 + 12 = 30 \end{array}\right\}

Greatest: \((x+3) = 18\)
7 \[ 4 \times 5 = 4 + 4 - 5 = 3 \]
\[ 6 \div \square = 6 + 6 - \square = 3 \]
\[ 12 - \square = 3 \]
\[ 12 = 3 + \square \]
\[ \boxed{9 = \square} \]

8 \[ \text{Ari + Barry} = 40 \]
\[ \text{Cara + Dara} = 30 \]
\[ \text{Erin + Ari} = 20 \]
\[ \boxed{Ari + Erin + (\text{Cara + Dara}) + \text{Barry} = 85} \]
\[ (20) + (30) + \text{Barry} = 85 \]
\[ 50 + \text{Barry} = 85 \]
\[ \text{Barry} = 35 \]

9 Let the weight of one ape = \(a\)
Let the weight of one bear = \(b\)
Let the weight of one cougar = \(c\)
Let the weight of one deer = \(d\)

\[ 9a = 4b \rightarrow 18a = 8b \]
\[ 8b = 15c \rightarrow 18a = 15c \rightarrow 6a = 5c \]
\[ 10c = 27d \rightarrow 12a = 27d \rightarrow 12a = 10c \]

\[ \boxed{9d = 4a} \]

Thus, 9 deer weigh the same as 4 apes
Let $A$ cents be the cost of one apple.
Let $B$ cents be the cost of one banana.

$5A + 3B = 570$ (Dan's order)
$3A + 5B = 470$ (Chris' order)

$\text{(Dan's order)} - \text{(Chris' order)} = 570 - 470$
$(5A + 3B) - (3A + 5B) = 100$
$5A + 3B - 3A - 5B = 100$

$2A - 2B = 100 \quad \rightarrow \quad \text{Divide both sides by 2}$

$\rightarrow \quad A - B = 50$

$\rightarrow \quad \text{an apple costs } 50 \text{ } \$ \text{ more}$

Mike is $n$ years old
Rick is $5n$ years old
Larry is $2n$ years old
Ed is $(\text{double Larry's } \text{& Mike's age}) - 30 = 6n - 30$ years old
Daniel is $(\text{Rick's age}) - 79 = 5n - 79$ years old

$(\text{sum of ages}) = 271$
$(n + 5n + 2n + 6n - 30 + 5n - 79) = 271$
$(19n - 109) = 271$
$19n = 380$
$n = 20 \quad \rightarrow \quad 5n - 79 = 5(20) - 79 = 21$
12. Let $x$ be the number of additional minutes talked.

$$25 + 3x = 40$$

$$3x = 15$$

$$x = 5$$

$5 + 3 = 8$ minutes

13. In the end, Maria has 3 marbles for every 1 that Juan has. So, out of 4 marbles, Maria has 3 and Juan has 1.
So, if there are 72 marbles, Maria has $\left(\frac{72}{4}\right)3 = 54$ marbles and Juan has $\left(\frac{72}{4}\right)1 = 18$ marbles.

<table>
<thead>
<tr>
<th>Total</th>
<th>Juan</th>
<th>Maria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marbles</td>
<td>72</td>
<td>18</td>
</tr>
</tbody>
</table>

In the end...

Before Juan gave Maria 12 marbles...

Before Juan gave Maria half his (at the start) 72 60 12

14. Let $N$ represent the original number.

$$10N - 0.1N = 33.66$$

$$9.9N = 33.66$$

$$N = 3.4$$

15. Members originally paid $90 + $60 = $150

Gifts cost $2 - $150 = $60.

If all members paid $2 to cover the gifts, there are $\frac{60}{20} = 30$ members.

If 30 members originally paid $150, each member paid $\frac{150}{30} = $5

To raise $90, there must be $\frac{90}{5} = 18$ boys.
Combinations Answers

1. 10 triangles
2. 13983816 combos
3. 80080 teams
4. a)133784560 hands
   b)1287 hands
5. 15 intersections
6. 4200 ways
7. 15 days without repeating pairs
8. 7315 teams

Work shown on the following pages
Answers have been circled
you're choosing 3 of 5 possible vertices

**Method 1:** Make a list of pairs of omitted vertices.

Example: AB means you formed \( \triangle CDE \)

- AB  BC  CD  DE
- AC  BD  CE
- AD  BE
- AE

\( \{ \text{you formed} \} \ 10 \text{ triangles} \)

**Method 2:** In-class, step-by-step method

- 5 choices for first vertex
- 4 choices for second vertex
- \( \times \ 3 \) choices for third vertex

\( \frac{5 \times 4 \times 3}{6} \) possible triangles, but that counts duplicates

How many different ways did you choose the same three vertices? Given 3 vertices...

- 3 choices for vertex one
- 2 choices for vertex two
- \( \times \ 1 \) choice for vertex three

\( \frac{3 \times 2 \times 1}{6} \) times

\( 60/6 = 10 \text{ triangles} \)
Method 3: Combination formula

\[ n = 5, \ r = 3 \]

\[
\frac{5!}{3!(5-3)!} = \frac{5!}{3! \times 2!} = \frac{120}{12} = 10 \text{ triangles}
\]

Method 1: In-class, step-by-step method

Lottery = order doesn't matter! → Combination

How many different ways did you choose the same six numbers? Give six numbers.

49 choices for #1
48 choices for #2
47 choices for #3
46 choices for #4
45 choices for #5
44 choices for #6

\[ \times 44 \text{ choices for } #6 \]

100,683,475,200 combinations

2 choices for #5
3 choices for #4
4 choices for #3
5 choices for #2
6 choices for #1

\[ \times 1 \text{ choices for } #6 \]

\[ \frac{100,683,475,200}{720} = 139,838,160 \text{ combinations} \]

Method 2: Combination formula

\[ n = 49, \ r = 6 \]

\[
\frac{49!}{6!(49-6)!} = \frac{1}{720} \times \frac{49!}{6! \times 43!} = \frac{49 \times 48 \times 47 \times 46 \times 45 \times 44}{720} = \frac{139,838,160}{720} \text{ combinations}
\]
Choosing 3 of 12 boys and 3 of 14 girls

Method 1: In class, step-by-step method

12 choices for boy one
11 choices for boy two
× 10 choices for boy one
\[ \frac{1320 \text{ combinations of boys}}{6} \]
14 choices for girl one
13 choices for girl two
× 12 choices for girl three
\[ \frac{2184 \text{ combinations of girls}}{6} \]

Eliminating Duplicate Combinations

3 choices for boy one
2 choices for boy two
× 1 choice for boy three
6 times
\[ \frac{220 \times 364}{6} = \text{80080 teams} \]

Method 2: Combination Formula

Boys: \( n = 12, \ r = 3 \)
\[ \frac{12!}{3!(12-3)!} \]
\[ \frac{12!}{3! \times 9!} \]
\[ \frac{1}{6} \times \frac{12!}{9!} = \frac{12 \times 11 \times 10}{6} = 220 \]

Girls: \( n = 14, \ r = 3 \)
\[ \frac{14!}{3!(14-3)!} \]
\[ \frac{14!}{3! \times 11!} \]
\[ \frac{1}{6} \times \frac{14!}{11!} = \frac{14 \times 13 \times 12}{6} = 364 \]
\[ 220 \times 364 = \text{80080 teams} \]
4 a) Method 1: In class, step-by-step method

52 choices for card one
51 choices for card two
50 choices for card three
49 choices for card four
48 choices for card five
47 choices for card six
46 choices for card seven

How many different ways did you choose the seven cards? Given seven cards...

52 choices for card #1
51 choices for card #2
50 choices for card #3
49 choices for card #4
48 choices for card #5
47 choices for card #6
46 choices for card #7

674,274,18,2400 hands

Method 2: Combination formula

\[ n = 52, \ r = 7 \]

\[ \frac{52!}{7!(52-7)!} = \frac{1 \times 52!}{7! \times 45!} = \frac{52 \times 51 \times 50 \times 49 \times 48 \times 47 \times 46}{5040 	imes 45!} = 1,337,845,600 \text{ hands} \]

b) Using the Combination formula

\[ n = 13, \ r = 5 \]

\[ \frac{13!}{5!(13-5)!} = \frac{13!}{5! \times 8!} = \frac{13 \times 12 \times 11 \times 10 \times 9}{120 	imes 8!} = 1,287 \text{ collections} \]
5 Count them! No need to use the formulas.

One chord \( \rightarrow \) at most 5 points of intersection

But be careful of duplicates! Chord #1 will intersect chord #2 at the same place chord #2 intersects chord #1

\[
\frac{(6 \times 5)}{2} = 15 \text{ intersections}
\]

6 Choose 1 of 10 drinks \( \rightarrow n = 10, r = 1 \)

choose 1 of 5 appetizers \( \rightarrow n = 5, r = 1 \)

choose 1 of 6 main courses \( \rightarrow n = 6, r = 1 \)

choose 2 of 8 sides \( \rightarrow n = 8, r = 2 \)

choose 1 of 5 desserts \( \rightarrow n = 5, r = 1 \)

\[
\frac{10!}{1!(10-1)!} \times \frac{5!}{1!(5-1)!} \times \frac{6!}{1!(6-1)!} \times \frac{8!}{2!(8-2)!} \times \frac{5!}{1!(5-1)!} \\
\frac{10!}{9!} \times \frac{5!}{4!} \times \frac{6!}{5!} \times \frac{8!}{1440} \times \frac{5!}{4!} \\
10 \times 5 \times 6 \times 28 \times 5
\]

4200 ways
6 members: A, B, C, D, E, F → order doesn't matter

**Method 1:** List possible pairs of students

AB  BC  CD  DE  EF
AC  BD  CE  DF
AD  BE  CF
AE  BF
AF

{15 pairs} → 15 days without repeating pairs

**Method 2:** Combination formula

\[ n = 6, \ r = 2 \]

\[ \frac{6!}{2!(6-2)!} = \frac{6!}{2! \times 4!} = \frac{720}{48} = 15 \text{ days without repeating pairs} \]

8 students, choosing 4 → order doesn't matter

\[ n = 22, \ r = 4 \]

\[ \frac{22!}{4!(22-4)!} = \frac{22!}{4! \times 18!} = \frac{1}{24} \times \frac{22!}{18!} = \frac{22 \times 21 \times 20 \times 19}{24} = 7315 \text{ teams} \]

This is choosing why teams hard
Permutations Answers

1.
   a. 6 ways
   b. 360 ways
   c. 40320 ways
   d. 12 ways
   e. 399147985716480000 ways

2. 60 different numerals
3. 120 ways
4. 28 arrangements
5. 1260 different signals
6. 210 different keys
7. 18 meals
8. 1/649740

Work shown on the following pages
Answers have been circled
1. a) **DOG** \( n = 3, \ r = 3, \ n = r \)
   
   number of permutations = \( n! = 3! = 6 \)

   b) **KITTEN** \( n = 6, \ r = 6, \ n = r \)
   
   and 2 of them are the same \( \rightarrow \rho! = 2! \)
   
   \[
   \text{number of permutations} = \frac{n!}{\rho!} = \frac{6!}{2!} = 360
   \]

   c) **MATHCLUB** \( n = 8, \ r = 8, \ n = r \)
   
   number of permutations = \( n! = 8! = 40320 \)

   d) **WCMC** \( n = 4, \ r = 4, \ n = r \)
   
   and 2 of them are the same \( \rightarrow \rho! = 2! \)
   
   \[
   \text{number of permutations} = \frac{n!}{\rho!} = \frac{4!}{2!} = 12
   \]

   e) **FAIRHAVEN MIDDLE SCHOOL** \( n = 21, \ r = 21, \ n = r \)
   
   and we have 2 A's, 2 I's, 2 H's, 2 E's, 2 O's, 2 L's, and 2 O's \( \rightarrow \rho! = (2!)(2!)(2!)(2!)(2!)(2!)(2!) \)
   
   \[
   \text{number of permutations} = \frac{21!}{(2!)(2!)(2!)(2!)(2!)(2!)(2!)} = \frac{21!}{128} = 399147985716480000
   \]

2. **4, 4, 5, 5, 7** \( n = 6, \ r = 6, \ n = r \)
   
   and we have 2 4's and 3 5's \( \rightarrow \rho! = (2!)(3!) \)
   
   \[
   \text{number of permutations} = \frac{6!}{(2!)(3!)} = 60
   \]
3. 5 people → n = 5 \{ n = r \\
5 seats → r = 5 \\
number of permutations = n! = 5! = 120

4. 6 Golden Delicious (GD), 2 Bartlett Pear (BP)
   → GD GD GD GD GD BP BP
   n = 8, r = 8, n = r
and we have 6 GD's and 2 BP's → \( \rho! = \frac{8!}{(6!)(2!)} \)
number of permutations = \( \frac{n!}{\rho!} = \frac{8!}{(6!)(2!)} = 28 \)

5. nine flags → n = 9, r = 9, n = r
and we have 3 reds, 2 blues, and 4 greens
   → \( \rho! = (3!)(2!)(4!) \)
number of permutations = \( \frac{9!}{(3!)(2!)(4!)} = \frac{9!}{288} = 1260 \)

6. 4 True (T), 6 False (F)
   → T T T T F F F F F F
   n = 10, r = 10, n = r
and we have 4 T's and 6 F's → \( \rho! = (4!)(6!) \)
number of permutations = \( \frac{10!}{(4!)(6!)} = \frac{10!}{17280} = 210 \)
7 choose 1 of 3 meats \( \to n = 3, \ r = 1 \)
choose 1 of 2 vegetables \( \to n = 2, \ r = 1 \)
choose 1 of 3 drinks \( \to n = 3, \ r = 1 \)

order doesn't matter \( \to \) combination problem

Just to refresh your memory ...

\[
\text{combination formula} = \binom{n}{r} = \frac{n!}{(r!) \cdot (n-r)!}
\]

meats \( \times \) vegetables \( \times \) drinks

\[
\frac{3!}{(1!) \cdot (3-1)!} \times \frac{2!}{(1!) \cdot (2-1)!} \times \frac{3!}{(1!) \cdot (3-1)!}
\]

\[
\frac{3!}{2!} \times \frac{2!}{1!} \times \frac{3!}{2!}
\]

\[
3 \times 2 \times 3
\]

8 18 meals

8 probability & choosing an A, k, Q, J, or 10 \( \to \frac{20}{52} \)
probability of choosing one of the four remaining necessary cards \( \to \frac{4}{51} \)

probability of choosing one of the three remaining necessary cards \( \to \frac{3}{50} \)
probability of choosing one of the two remaining necessary cards \( \to \frac{2}{49} \)
probability of choosing the last necessary card \( \to \frac{1}{48} \)

\[
\left( \frac{20}{52} \right) \left( \frac{4}{51} \right) \left( \frac{3}{50} \right) \left( \frac{2}{49} \right) \left( \frac{1}{48} \right) = \frac{1}{649,740}
\]
Probability and Statistics Answers

1. 1440 possible orders
2. 48
3. 0.17
4. $45
5. Page 22
6. 12/27 or 4/9

Work shown on the following pages
Answers have been circled
1 Tests: T₁, T₂, T₃, T₄, T₅, S, I
S and I have to be paired together, either like SI or IS.

\[
\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\end{array}
\]
: Test order

6 choices for SI or IS

2 ways to arrange S and I: SI, IS

Then we place the other five team tests:

5 options for T₁,
4 options for T₂,
3 options for T₃
2 options for T₄
1 option for T₅

\[
\left\{ \begin{array}{c}
6 \times 2 \times 5 \times 4 \times 3 \times 2 \times 1 \\
1440
\end{array} \right. 
\]

2 \\

average: 28 44

Method 1:
We could also write this serious like this:

\[
\begin{array}{c}
28 \\
44
\end{array}
\]

It takes two equal "jumps" to get from 28 to 44.

44 - 28 = 16, so each jump is 8. It takes

half a jump, 4, to go from 44 to the largest
multiple, so the largest multiple is 48.
2 continued...

Method 2:

The sum of the first two multiples is $28(2) = 56$
The sum of the second two multiples is $44(2) = 88$.

Greatest common factor of 56 and 88 is 8. So we know the "same number" is 8, because if Abby's list of numbers continued 56 and 88 would both be on it.

Multiples of 8: 8 16 24 32 40 48 56 ...

Average to 28 Average to 44

→ the largest multiple on Abby's list is 48.

3 Think combinations!

Step 1: choose 2 of 4 identical cards for your pair
→ $n = 4$, $r = 2$

Step 2: choose the face value of your pair
→ $n = 13$, $r = 1$

Step 3: choose the face value of your third card
→ $n = 12$, $r = 1$

Step 4: choose the suit of your third card
→ $n = 4$, $r = 1$

Results from using the formula

Step 1: $\frac{4!}{2!(4-2)!} = 6$

Step 2: $\frac{13!}{1!(13-1)!} = \frac{13!}{12!} = 13$

Step 3: $\frac{12!}{1!(12-1)!} = \frac{12!}{11!} = 12$

Step 4: $\frac{4!}{1!(4-1)!} = \frac{4!}{3!} = 4$

Number of ways to choose a one-pair: 3744

Number of ways to choose a 3-card poker hand: $\frac{52!}{3!(52-3)!} = \frac{52!}{3!} = \frac{52 \times 51 \times 50}{3!} = \frac{22100}{6} = 3744$

Number of ways to choose 3744/22100 \approx 0.17
Method 1: Using algebra

Carol: \(3c + 2v = 31\)

Gabriel: \(4c + 3v = 43\)

\[\begin{align*}
3c + 2v &= 31 \\
\quad &\quad \rightarrow 2v = 31 - 3c \\
\quad &\quad \rightarrow v = \frac{31 - 3c}{2}
\end{align*}\]

\[\begin{align*}
4c + 3v &= 43 \\
\quad &\quad \rightarrow 4c + 3\left(\frac{31 - 3c}{2}\right) = 43
\end{align*}\]

\[\begin{align*}
\quad &\quad \rightarrow 4c + \frac{93 - 9c}{2} = 43 \\
\quad &\quad \rightarrow \frac{-5c}{2} = \frac{-7}{2} \\
\quad &\quad \rightarrow c = 7
\end{align*}\]

\[\begin{align*}
3c + 2v &= 31 \\
\quad &\quad \rightarrow 3(7) + 2v = 31 \\
\quad &\quad \rightarrow 2v = 10 \\
\quad &\quad \rightarrow v = 5
\end{align*}\]

Bridget: \(5c + 2v = 5(7) + 2(5) = 35 + 10 = \$45\)

Method 2: Using logical reasoning

Gabriel has one more consonant and one more vowel than Carol. This implies that one consonant and one vowel cost \(43 - 31 = \$12\). 2 consonants and 2 vowels cost twice as much, \(\$24\). But Carol has 3 consonants and 2 vowels, so that one extra consonant costs \(31 - 24 = \$7\). So, 3 consonants in Carol's name cost \(3(7) = \$21\), which implies that the two vowels cost \(31 - 21 = \$10\). So each vowel costs \$5.

So, Bridget, with 5 consonants and 2 vowels, costs \((5 \times 7) + (2 \times 5) = 35 + 10 = \$45\)

Let \(p\) = the starting page number.

The following pages thus equal \(p + 1, p + 2, p + 3\).

We are given \(p + (p + 3) = 47\).

So, \(p + (p + 3) = 47 \rightarrow 2p + 3 = 47 \rightarrow 2p = 44 \rightarrow p = 22\)
<table>
<thead>
<tr>
<th>Possible letter combinations: (Cate, Amy, Brett)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZZ, Z</td>
</tr>
<tr>
<td>ZT, Z</td>
</tr>
<tr>
<td>TZZ</td>
</tr>
</tbody>
</table>

\[
\frac{12}{27} = \frac{4}{9} \quad \text{is the probability that Cate's letter is different from both Amy's and Brett's.}
\]
Venn Diagrams Answers

1. 
   a. 2 students 
   b. 38 students 
   c. 3/5 or 60%
2. 8 students 
3. 7 students 
4. 225 students 
5. 30% 
6. 5 patrons 
7. 83 students 
8. 7 students 

Work shown on the following pages 
Answers have been circled
a) 2 students
b) 9 + 5 + 24 = 38 students
c) \( \frac{24}{40} = \frac{3}{5} = 60\% \)

\[ x = 50 - 16 - 2 - 24 \]
\[ x = 8 \text{ students} \]
3

Tennis

(x)

Chess

(15-x)

(11-x)

3 + (15-x) + x + (11-x) = 25
3 + 15 - x + x + 11 - x = 25
29 - x = 25
x = 4

Chess = 11 - x = 11 - 4
= 7 students

4

Band

Sports

(85-60)

(200-60)

25

60

140

25 + 60 + 140 = 225 students

"Either band or sports" means you include the students who do only band, only sports, and both band and sports!

5

Owes a computer

(x)

Is in Band

(80-x)

(40-x)

(320 - 25 - 60 - 140) = 95

Let's say there are 100 mathletes ...

(80-x) + x + (40-x) + 10 = 100
80 - x + x + 40 - x + 10 = 100
-x = -30

x = 30 students = 30/100 = 30%
Has a Dog: (14-4-0-3) 7
Has a Cat: (10-4-0-1) 5
Has a Fish: (5-3-0-1) 1

26 - 7 - 4 - 5 - 3 - 0 - 1 - 1 = x
5 = x \rightarrow 5 \text{ patrons}

Basketball: (34-6-3 -17) 9
Basketball: (39-7-5 -3) 24
Basketball: (33-17-3-7) 6

Softball: (34-6-3 -19) 17
Softball: (8-3) 5
Softball: (39-7-5 -3) 10

Volleyball: (20-3) 3
Volleyball: (10-3) 7
Volleyball: (33-17-3-7) 12

9 + 5 + 24 + 17 + 3 + 7 + 6 + 12 = 83 \text{ students}
Latin = 11
11 = 5 + 2 + 3 + SL
1 = SL

Spanish = 16
16 = 8 + SL + 3 + FS
16 = 8 + 1 + 3 + FS
4 = FS

French = 16
16 = x + FS + 3 + 2
16 = x + 4 + 3 + 2

7 students ← 7 = x
Base Conversion Answers

1.
   a. 6 ways
   b. 360 ways
   c. 40320 ways
   d. 12 ways
   e. 399147985716480000 ways
2. 60 different numerals
3. 120 ways
4. 28 arrangements
5. 1260 different signals
6. 210 different keys
7. 18 meals
8. 1/649740

Work shown on the following pages
Answers have been circled
1. a) **DOG**  \( n = 3, \ r = 3, \ n = r \)
   
   Number of permutations = \( n! = 3! = 6 \)

b) **KITTEN**  \( n = 6, \ r = 6, \ n = r \)
   
   And 2 of them are the same \( \Rightarrow p! = 2! \)
   
   Number of permutations = \( \frac{n!}{p!} = \frac{6!}{2!} = 360 \)

c) **MATHCLUB**  \( n = 8, \ r = 8, \ n = r \)
   
   Number of permutations = \( 8! = 40320 \)

d) **WCMC**  \( n = 4, \ r = 4, \ n = r \)
   
   And 2 of them are the same \( \Rightarrow p! = 2! \)
   
   Number of permutations = \( \frac{n!}{p!} = \frac{4!}{2!} = 12 \)

e) **FAIRHAVENMIDDLE SCHOOL**  \( n = 21, \ r = 21, \ n = r \)
   
   And we have 2 A's, 2 I's, 2 H's, 2 E's, 2 O's, 2 L's, and 2 O's \( \Rightarrow p! = (2!)(2!)(2!)(2!)(2!)(2!) \)
   
   Number of permutations = \( \frac{21!}{(2!)(2!)(2!)(2!)(2!)(2!)} \)
   
   \( = \frac{21!}{128} \)
   
   \( = 3991479857164800000 \)

2. \( 4,4,5,5,5,7 \)  \( n = 6, \ r = 6, \ n = r \)

   And we have 2 4's and 3 5's \( \Rightarrow p! = (2!)(3!) \)
   
   Number of permutations = \( \frac{6!}{(2!)(3!)} = 60 \)
3 5 people $\rightarrow n=5 \Rightarrow n=r$ 
5 seats $\rightarrow r=5$ 
number of permutations = $n! = 5! = 120$

4 6 Golden Delicious (GD), 2 Bartlett Pear (BP) 
$\rightarrow$ GD GD GD GD GD GD BP BP 
$n=8$, $r=8$, $n=r$ 
and we have 6 GD's and 2 BP's $\rightarrow r! = (6!)(2!)$ 
number of permutations = $\frac{n!}{r!} = \frac{8!}{(6!)(2!)} = 28$

5 nine flags $\rightarrow n=9$, $r=9$, $n=r$ 
and we have 3 reds, 2 blues, and 4 greens 
$\rightarrow r! = (3!)(2!)(4!)$ 
number of permutations = $\frac{9!}{(3!)(2!)(4!)}$ 
$= \frac{9!}{288} = 1260$

6 4 True (T), 6 False (F) 
$\rightarrow$ T T T T F F F F F F 
$n=10$, $r=10$, $n=r$ 
and we have 4 T's and 6 F's $\rightarrow r! = (4!)(6!)$ 
number of permutations = $\frac{10!}{(4!)(6!)}$ 
$= \frac{10!}{17280} = 210$
7 choose 1 of 3 meats \(\rightarrow n = 3, r = 1\)
choose 1 of 2 vegetables \(\rightarrow n = 2, r = 1\)
choose 1 of 3 drinks \(\rightarrow n = 3, r = 1\)

Order doesn't matter \(\Rightarrow\) combination problem
Just to refresh your memory ...

Combination formula: \(\binom{n}{r} = \frac{n!}{(r!)(n-r)!}\)

\[
\begin{align*}
\frac{3!}{(1!)(3-1)!} \times \frac{2!}{(1!)(2-1)!} \times \frac{3!}{(1!)(3-1)!} \\
\frac{3!}{2!1!} \times \frac{2!}{1!} \times \frac{3!}{2!} \\
3 \times 2 \times 3
\end{align*}
\]

18 meals

8 Probability of choosing an A, K, Q, J, or 10: 20/52
Probability of choosing one of the four remaining necessary cards: 4/51

Probability of choosing one of the three remaining necessary cards: 3/50

Probability of choosing one of the two remaining necessary cards: 2/49

Probability of choosing the last necessary card: 1/48

\[
\left(\frac{20}{52}\right) \left(\frac{4}{51}\right) \left(\frac{3}{50}\right) \left(\frac{2}{49}\right) \left(\frac{1}{48}\right) = \frac{1}{6,497,400}
\]
Logic Answers

1. Flavors (from bottom to top):
   - Strawberry, Bubblegum, Blueberry, Vanilla, Chocolate
2. 80 teachers
3. 5 more brothers than sisters
4. 13 votes
5. 3715
6. 15 three-digit numbers
7. Thursday
8. \((m \times n) - 1\) steps

BONUS.
   You don’t have the basket anymore. See following pages for more details.

Work shown on the following pages
Answers have been circled
1. **Bottom Flavor**: Strawberry (only one with 10 letters). Vanilla must be directly below chocolate but directly above blueberry (vanilla touches both chocolate and blueberry and we know the vanilla is below the chocolate).

   Bubblegum is directly below the blueberry (we know it is below the vanilla, but not directly below because the blueberry is directly below).

   **Flavors (from bottom to top)**:
   
   Strawberry, Bubblegum, Blueberry, Vanilla, Chocolate.

2. Total class slots on all schedules = 1600 students × 5 classes = 8000

   8000 class slots ÷ 25 students per class = 320 classes daily.

   320 classes daily ÷ 4 per teacher = 80 teachers.

3. Let $M =$ number of brothers
   
   Let $F =$ number of sisters.

   There are $4 + F$ males.

   Sara has $F - 1$ sisters.

   
   \[ 4 + F - (F - 1) = 5 \]

   Sara has 5 more brothers than sisters.
4. A gets 50 votes \{ 45 votes left for B, C, D, E \\
\Rightarrow \text{ E gets 5 votes} \\

\Rightarrow \text{ Assume all four middle candidates received close to average} \\
\Rightarrow \text{ Average: } 45/4 = 11.25 \approx 12

Suppose B got 13, C got 12, D got 11, E got 10 votes \\
\text{ This violates Fact #5.} \\
\text{ However, if E got one vote less and B got one vote more:} \\
\Rightarrow \text{ B got 13 votes} \\
\Rightarrow \text{ C got 12 votes} \\
\Rightarrow \text{ D got 11 votes} \\
\Rightarrow \text{ E got 9 votes} \\
\Rightarrow \text{ B received at least 13 votes}

5. Divisible by 5 \Rightarrow \text{ the ones digit is a 5: ???5} \\
\text{ The 7 is next to the 1, but not the 5} \Rightarrow \text{ 715, 275} \\
\text{ The 3 is next to the 7, but not the 5} \Rightarrow \text{ 3715}

6. Sum of tens digit and ones digit = 9 \\
\text{ It's even} \Rightarrow \text{ ?18, ?90, ?72, ?54, ?36} \\
\text{ Divisible by 3 (the sum of the digits is a multiple of 3)} \\
\Rightarrow \text{ ?1, ?3, ?6, or 9}

\text{ Possible #'s: 318, 390, 372, 354, 336, 618, 690, 672, 654,} \\
\text{ 636, 918, 990, 972, 954, 936} \Rightarrow \text{ 15 three-digit #'}s
"Tomorrow I will lie"

<table>
<thead>
<tr>
<th>Speaker</th>
<th>Sun</th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thurs</th>
<th>Fri</th>
<th>Sat</th>
</tr>
</thead>
<tbody>
<tr>
<td>John</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>Minnie</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

"Truth teller": "Tomorrow I will lie" → True

"Liar": "Tomorrow I will lie" → False

only two consecutive days on which the truth status changes for both speakers is from Thursday to Friday. → The only day on which both could say "Tomorrow I will lie" is Thursday.

8 By breaking an existing piece horizontally or vertically, you increase the total number of pieces by one.

You have one piece, so you need \((m \times n) - 1\) steps to get \(m \times n\) pieces

**Bonus**

You give an apple to each of your first nine friends, and the basket with the last apple in it to your 10\(^{th}\) friend. Each friend has an apple and there is still one apple in the basket, but you don’t have the basket anymore.
Review Answers

1  1728 in$^3$
2  x = 9
3  20/61
4  24 cats
5  135 pancakes
6  7/50
7  approximately 13.16 units
8  70 cents more
9  7 whole numbers
10 1.94
11 17 minutes
12 Jim is 32 years old
13 $53$
14 363
15 7.5 square units
16 2/15
17 A = 2044
18 45\pi
19 34°
20 1/649740

Work shown on the following pages
Answers have been circled
Factors of 108: (2,54), (3,36), (4,27), (6,18), (9,12)

Possible y-values (#'s that go into 144 evenly) are circled

Using the possible y-values, find possible z-values.

Plug the possible z-values in with the corresponding x-values

("corresponding" refers to the pairs of factors of 108, i.e.

if \( y = 6 \) (the corresponding x-value is 18) to \( zx = 192 \)

to see if that y-value works.

\[ y = 9 \quad \rightarrow \quad \text{corresponding x-value} = 12 \]
\[ 2y = 144 \quad \rightarrow \quad z = 16 \]
\[ zx = 192 \quad \rightarrow \quad 192/z = x \quad \rightarrow \quad 192/16 = 12 = x \]

\[ x = 12 \]
\[ y = 9 \]
\[ z = 16 \]

\[ \text{Volume} = (12)(9)(16) = 1728 \text{ in}^3 \]

2

\[ \text{m} = 4m + 3 \]

\[ 4(4(4(4x + 3) + 3) + 3) + 3 = 10239 \]

\[ 4(4(4(4x + 3) + 3) + 3) + 3 = 10239 \]

\[ 4(4(4(4x + 3) + 3) + 3) + 3 = 10239 \]

\[ 4(4(4(4x + 3) + 3) + 3) + 3 = 10239 \]

\[ 4(4(4(4x + 3) + 3) + 3) + 3 = 10239 \]

\[ 4(4(4(4x + 3) + 3) + 3) + 3 = 10239 \]

\[ 4(4(4(4x + 3) + 3) + 3) + 3 = 10239 \]
4 \(256x + 192 + 48 + 12 + 3\) + 3 = 10239
1024x + 768 + 192 + 48 + 12 + 3 = 10239
1024x + 1023 = 10239
1024 x = 9216

\[x = 9\]

3. \(xy\) is divisible by 3 if \(z\) is divisible by 3 where \(x + y = z\)

We know \(x = 2, 3, 4, 5, 6,\) or 7

\[y = 1, 2, 3, 4, 5, 6, 7, 8,\) or 9

\[z = 3, 6, 9, 12,\) or 15

Numbers that qualify:

21, 24, 27, 30, 33, 36, 39, 42, 45, 48,
51, 54, 57, 60, 63, 66, 69, 72, 75, 78

20 qualifying numbers

61 numbers between 20 and 80 inclusive

4. Let \(c\) = number of cats
Let \(b\) = number of birds

\[36 = c + b \rightarrow 36 - c = b\]

120 = 4c + 26

\[120 = 4c + 2(b)\]
\[120 = 4c + 72 - 2c\]
\[120 = 2c + 72\]
\[48 = 2c\]
\[24 = c\]

Annie has 24 cats
5. Volume of 12 in diameter, 1 in high pancake = \( \pi (6)^2 \cdot 1 = 36\pi \text{ in}^3 \)
15, 12 in diameter, 1 in high pancakes = \( 15 \cdot 36\pi = 540 \text{ in}^3 \)
Volume of 4 in diameter, 1 in high pancake = \( \pi (2)^2 \cdot 1 = 4\pi \text{ in}^3 \)
With \( 540\pi \text{ in}^3 \) you can make how many \( 4\pi \text{ in}^3 \) pancakes?
\[
\frac{540\pi \text{ in}^3}{4\pi \text{ in}^3} = \frac{135}{1} \text{ pancakes}
\]

6. Vowels: a, e, i, o, u
Numbers that begin with a vowel:
1, 8, 11, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29
14 numbers that begin with a vowel \( \Rightarrow \) \( \frac{14}{100} = \frac{7}{50} \)

100 numbers between 0 and 99 inclusive

7. 
\[
\frac{1}{2} (b_1 + b_2) h = 9
\]
\[
\left( \frac{13}{4} \right) (5^2) (3) = 9
\]
\[
\left( \frac{13}{4} \right) (5^2) = 3
\]
\[
\left( \frac{13}{3} \right) (5^2) = 12
\]
\[
\left( \frac{13}{3} \right) (5^4) = 144
\]

\( b_1 = 5 \)
\( b_2 = 25 \)
\( 5^4 = 48 \)
\( s \approx 2.632 \ldots \)

Perimeter = \( 2s + b_1 + b_2 \)
\[
= 2s + 5 + 25
= 5s
\approx 5(2.63)
\approx 13.16 \text{ units} \)
Let a cents be the cost of an apple
Let b cents be the cost of a banana

\[ 5a + 3b = 12.75 \quad \text{(Dan's order)} \]
\[ 3a + 5b = 11.35 \quad \text{(Chris' order)} \]

\[(\text{Dan's order}) - (\text{Chris' order}) = 12.75 - 11.35\]
\[(5a + 3b) - (3a + 5b) = 140 \]
\[5a + 3b - 3a - 5b = 140\]
\[2a - 2b = 140\]
\[a - b = 70\]
\[a = 70 + b \quad \rightarrow \text{an apple costs} \]
\[70 \text{¢ more than a banana}\]

If our number \((N)\) has only 3 factors that means it is a perfect square

Factors of \(N\) : 1, \(x\), \(N\)

where \(N\) is a perfect square and \(x\) is its square root.

Since there are no other factors we know \(x\) is prime.

\(N\) is a perfect square
\(x\) is a prime and \(x = \sqrt{N}\)

Possible \(x\)'s: 2, 3, 5, 7, 11, 13, 17

Corresponding \(N\)'s: 4, 9, 25, 49, 121, 169, 289 → 7 \(N\)'s that qualify

There are 7 whole numbers between 1 and 300 that have exactly three factors
\[ \frac{70}{36} \approx 1.94 \]

Call one costed \((33+5x)\) cents

Call two costed \((33+5y)\) cents

\[ 5x + 5y = 35 \]

\[ 3x + 5x + 3y + 5y = 101 \]

\[ 7 \text{ minutes over the 10 initial minutes} \]

\[ \frac{y}{x-y} \] where \(x\) is the larger of die one and die two and \(y\) is the smaller of die one and die two.
Let $D = \text{Dave's age}$, $J = \text{Jim's age}$, $B = \text{Bill's age}$

$D = 3J$  \hspace{1cm} $B = 3\left(\frac{1}{2}B\right) = \frac{3}{2}B$

$J = \frac{1}{5}B$

\[
64 = \frac{(B + J + D)}{3} = \frac{(B + \frac{1}{5}B + \frac{3}{2}B)}{3} = \frac{3B}{3} = B
\]

$B = 64$

$J = \frac{1}{5}B = \frac{1}{5}(64) = 32$  \hspace{1cm} Jim is 32 years old

Let $x = \text{the number of friends}$,

and let $3x$ represent $x$ friends paying $3$ each.

From the information given, we know: $6x + 5 = 8x + 11$

Through algebraic manipulation,

we get that $16 = 2x$ and thus that $8 = x$.

So, we know there are 8 friends.

Knowing this, you can find out $6x = 6(8) = 48$ and $8x = 8(8) = 64$.

We know that $48$ is $5$ too little and $64$ is $11$ too much, so:

$48 + 5 = 53 = 64 - 11$

Thus, the video game cost $53$. 
**Strategy:** determine how many of the first 300 numbers can't be used.

<table>
<thead>
<tr>
<th>&quot;Set&quot;</th>
<th>numbers that can’t be used</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 100</td>
<td>9, 19, 29, 39, 49, 59, 69, 79, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99</td>
</tr>
<tr>
<td>101 - 200</td>
<td>same numbers as above with a 1 in front</td>
</tr>
<tr>
<td>201 - 300</td>
<td>same numbers as the 1-100 set with a 2 in front</td>
</tr>
</tbody>
</table>

→ each set "looses" 19 numbers
→ lost 3(19) = 57 numbers overall
→ "refill" the list with the next 57 numbers (301 → 357)
→ eliminate 309, 319, 329, 339, 349
→ "refill" with 5 more numbers (358 → 362)
→ eliminate 359
→ "refill" with 1 more number: 300^{th} # = 363

---

If $x = 0$, $12x - 6y = 30$

$-6y = 30$

$y = -6$

If $y = 0$, $12x - 5y = 30$

$12x = 30$

$x = \frac{5}{2}$

Area = \frac{1}{2} \text{(base)} \cdot \text{(height)}

= \frac{1}{2} \left( \frac{5}{2} \right) (6)

= \frac{15}{2} \text{ units}^2

= 7.5 \text{ units}^2
Any number divisible by 15 is also divisible by 3 and 5.
Any number divisible by 5 ends in 0 or 5.
Any number divisible by 3 has digits that sum to a multiple of 3.

5 numbers \( n = 5 \)
choosing 3 \( r = 3 \)

\[
\frac{n!}{(n-r)!} = \frac{5!}{(5-3)!} = \frac{5!}{2!} = (5)(4)(3)
\]

= 60 possible 3-digit numbers

→ must end in 5 to be divisible by 5 (no option for the 3-digit number to end in 0)

→ \( xy5 \) is a 3-digit number that ends in 5 where \( x \) and \( y \) are the positive integers 1, 3, 7, or 9

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>3</th>
<th>3</th>
<th>3</th>
<th>7</th>
<th>7</th>
<th>7</th>
<th>9</th>
<th>9</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>7</td>
<td>9</td>
<td>1</td>
<td>7</td>
<td>9</td>
<td>1</td>
<td>3</td>
<td>9</td>
<td>1</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>is ( xy5 ) a multiple of 3?</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
</tbody>
</table>

8 3-digit number that are multiples of both 5 and 3

\[
\frac{8}{60} = \frac{2}{15}
\]
B is a multiple of 5 \[\rightarrow B \text{ ends in 0 or 5}\]

Multiples of 4 and 6 must be even \[\rightarrow B \text{ ends in 0}\]

A ends in 4

C ends in 6

The number formed by the last two digits of any multiple of 4 is divisible by 4 \[\rightarrow A \text{ ends in 04, 24, 44, 64, or 84}\]

Any multiple of 6 is also a multiple of 3 which means the sum of its digits is a multiple of 3

Starting at 2000...

<table>
<thead>
<tr>
<th>A</th>
<th>2004</th>
<th>2024</th>
<th>2044</th>
<th>2064</th>
<th>2084</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>2005</td>
<td>2025</td>
<td>2045</td>
<td>2065</td>
<td>2085</td>
<td>...</td>
</tr>
<tr>
<td>C</td>
<td>2006</td>
<td>2026</td>
<td>2046</td>
<td>2066</td>
<td>2086</td>
<td>...</td>
</tr>
</tbody>
</table>

Is C a multiple of 3

No, No, Yes, No, No, ...

Smallest possible value of A = 2044
\[ s^2 + s^2 = 72 \text{ units}^2 \]
\[ 2s^2 = 72 \text{ units}^2 \]
\[ s^2 = 36 \text{ units}^2 \]
\[ s = 6 \text{ units} \]

\[ c^2 = 6^2 + 12^2 \]
\[ c^2 = 180 \]
\[ c = \sqrt{180} = \text{diameter of circle} \]
\[ \frac{\sqrt{180}}{2} = \text{radius of circle} \]

Area = \( \pi \left( \frac{180}{2} \right)^2 \)
= \( \pi \left( \frac{180}{4} \right) = 45\pi \)

19
360° for the whole clock
Hour hand angle = \( \frac{360°}{12 \text{ hr}} = 30° \text{ per hour} = 0.5° \text{ per minute} \)
Minute hand angle = \( \frac{360°}{60 \text{ min}} = 6° \text{ per minute} \)

If it is \( H : M \) ...
\[ \frac{1}{2} (60H + M) - 6M \]

It is 7:32 ...
\[ \frac{1}{2} (60(7) + 32) - 6(32) \]
\[ \frac{1}{2} (452) = 192° \]
\[ 226 - 192 \]
\[ 34° \]
20 Probability of choosing an A, K, Q, J, or 10 \[ \frac{20}{52} \]
Probability of choosing one of the four remaining necessary cards \[ \frac{4}{51} \]
Probability of choosing one of the three remaining necessary cards \[ \frac{3}{50} \]
Probability of choosing one of the two remaining necessary cards \[ \frac{2}{49} \]
Probability of choosing the last necessary card \[ \frac{1}{48} \]

\[
\left( \frac{20}{52} \right) \left( \frac{4}{51} \right) \left( \frac{3}{50} \right) \left( \frac{2}{49} \right) \left( \frac{1}{48} \right) = \frac{1}{649,740}
\]
Mock Tests

Mental Math Sections
1. What is the product of 13 and 15?

2. A bag is filled with two yellow marbles and 6 red marbles. When one marble is drawn from the bag, what is the probability, as a percent, that it is red?

SWITCH TEAM MEMBERS

3. Start with nine, then add six, then subtract three, then divide by two. What number do you get?

4. What is two-thirds of 36, minus one-half of 14?

SWITCH TEAM MEMBERS

5. If you start reading a book at 10:55 and it takes four hours and 15 minutes to read the book, at what time will you be finished?

6. Find seven squared and add three squared.

SWITCH TEAM MEMBERS

7. How many thirds are there in twelve?

8. What is the surface area of a cube with the sides of 4?
Mental Math

1. If a rectangle has an area of 64 and the width is four times the length, what is the perimeter?
2. What is half-a-dozen plus one-third of half-a-dozen?

SWITCH TEAM MEMBERS
3. What is the square of the difference of 23 and 9?
4. One-zero in base a is what number in base ten?

SWITCH TEAM MEMBERS
5. Willow eats half of her Valentines candy on February 15th and half of what's left on February 16th. If she has only 6 pieces of candy left on the seventeenth, how many pieces did she start with?
6. If it takes Maddy two and a half hours to drive 95 miles, how fast is she going in miles per hour?

SWITCH TEAM MEMBERS
7. What is the product of 42 and 19?
8. What is 90% of 90?
Whatcom County Math Championship
2014 Mock Test

Mental Math

1. If the area of a circle is thirty-six pi, what is the diameter of the circle?
2. What is the sum of the first seven odd numbers?

SWITCH TEAM MEMBERS

3. If you start with the number eighteen, double the number, then subtract one-third of the result, what do you get?
4. What is the difference between six squared and eleven squared?

SWITCH TEAM MEMBERS

5. How many diagonals can be drawn in a regular hexagon?
6. If I roll two six-sided dice and add the results, what is the largest odd result I could get?

SWITCH TEAM MEMBERS

7. If the sum of two numbers is forty-three and one of the numbers is seventeen, what is the other number?
8. What is seventy percent of sixty?
Whatcom County Math Championship
2015 Mock Test

Mental Math

1. If you roll two six sided dice and add the results together, how many different outcomes are there?
2. If it is noon right now, what time will it be in 110 hours?

SWITCH TEAM MEMBERS

3. What is the greatest number of circles of radius one that can fit into a square of area 36?
4. What is half of one plus six-dozen of another one?

SWITCH TEAM MEMBERS

5. How many degrees does the minute hand rotate from midnight to 2:45 AM?
6. What is 2,015 minus 5,102?

SWITCH TEAM MEMBERS

7. How many diagonals are there in a pentagon?
8. A regular hexagon with side length 6 is cut in half along a line of symmetry. What is the perimeter of the resulting quadrilateral?
Mock Test

Answers
Individual Answers
1. $7
2. 25 prime numbers
3. 3/4
4. 2/3
5. rectangle
6. 153
7. $12
8. 64
9. 36°
10. 0
11. 8
12. 60

Algebra Answers
1. 95
2. 49192 seconds
3. 111010
4. 381

Geometry Answers
1. 24 units²
2. 60 rectangles
3. 8 units²
4. 130°

Probability & Statistics Answers
1. 70
2. 1/3
3. 9
4. 5/16

Potpourri Answers
1. –8
2. half of a third of a fifth
3. 82.5 cm²
4. 120
1. Let \( w \) be the cost of a widget.
   Let \( t \) be the cost of a thingamajig.

\[
4w + 3t = 29 \\
5w + 2t = 24
\]

\[
\rightarrow t = 12 - \frac{5}{2}w \\
\rightarrow 4w + 3\left(12 - \frac{5}{2}w\right) = 29 \\
\rightarrow 4w + 36 - \frac{15}{2}w = 29 \\
\rightarrow \frac{-7}{2}w = -7 \\
\rightarrow w = 2
\]

\[
5w + 2t = 24 \\
\rightarrow 5(2) + 2t = 24 \\
\rightarrow 10 + 2t = 24 \\
\rightarrow 2t = 14 \\
\rightarrow t = 7
\]\n
\( \rightarrow \text{one thingamajig costs } \$7 \)

2. Primes less than 100:
   2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47,
   53, 59, 61, 67, 71, 73, 79, 83, 89, 97

\( \rightarrow 25 \text{ prime numbers} \)

3. \[
\begin{align*}
\text{Area of outside circle: } & 100\pi \\
\text{Area of inner circle: } & 25\pi \\
\text{Area of shaded region: } & 75\pi
\end{align*}
\]

\[
\frac{75\pi}{100\pi} = \frac{3}{4}
\]
Factors of 6: 1, 2, 3, 6

Possible dice rolls: 1, 2, 3, 4, 5, 6

\[
\text{Probability of rolling a factor of 6: } \frac{4}{6} = \frac{2}{3}
\]

By connecting the midpoints of each side of a rhombus, you form a rectangle.

\[
5A = 5! + 4! + 3! + 2! + 1!
= 120 + 24 + 6 + 2 + 1
= 153
\]

Let \( x \) be the original cost of the book.

Jane bought the book for \( 20 - 9.92 = $10.08 \)

\[
10.08 = x(0.9) + (x(0.8))(0.05)
10.08 = 0.8x + 0.04x
10.08 = 0.84x
x = 12
\]

The book originally cost \$12.

Perfect squares: 0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, ...

Perfect cubes: 0, 1, 8, 27, 64, 125, ...

\[
\rightarrow 64
\]
9. If \( \triangle ABC \) is isosceles that means \( AB = BC \) and \( \angle BAC = \angle BCA \). Since a triangle's interior angles sum to be 180°, we know \( \angle ABC \) has to be the angle that equals 108° (note: my figure is not drawn to scale). So \( \angle BAC \) and \( \angle BCA \) must equal \( \frac{180 - 108}{2} = 36° \)

10. \((a-b)^2 \neq (b-a)^2 = ((a-b)^4 - (b-a)^4)\\=(a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4) - (a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4)\\= a^4 - a^4 - 4a^3b + 4a^3b + 6a^2b^2 - 6a^2b^2 - 4ab^3 + 4ab^3 + b^4 - b^4\\= 0\\

11. \((\frac{16}{4})^3 = x^2\\\(4)^3 = x^2\\64 = x^2\\x = 8\\

12. \((12)^2 + x^2 = (13)^2\\144 + x^2 = 169\\x^2 = 25\\x = 5\\\text{Area} = \frac{1}{2}(24)(5) = 60
1

)(((2*3)*3)*3)*3)

(((2(2+1)*3)*3)*3)

(((5*3)*3)*3)

(((2(5+1)*3)*3)*3)

(((11*3)*3)*3)

(((2(11+1)*3)*3)*3)

((23*3)*3)

((2(23+1)*3)*3)

((47*3)*3)

(2(47+1) = 95

2

6:32.24 → 6:33.0 → 0:00.36
6:33.0 → 7:00.0 → 0:27.0
7:00.0 → 12:00.0 → 5:00.0
12:00.0 → 8:12.16 → 8:12.16 +
13:39.52

13 hours = 780 minutes = 46800 seconds
39 minutes = 2340 seconds
52 seconds

46800 + 2340 + 52 = 49192 seconds
3 \[58_{10} \rightarrow \text{binary}\]

<table>
<thead>
<tr>
<th>(base) power</th>
<th>2^5</th>
<th>2^4</th>
<th>2^3</th>
<th>2^2</th>
<th>2^1</th>
<th>2^0</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>32</td>
<td>16</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

\[58 / 32 = 1 \text{ remainder } 26\]
\[26 / 16 = 1 \text{ remainder } 10\]
\[10 / 8 = 1 \text{ remainder } 2\]
\[2 / 4 = 0 \text{ remainder } 2\]
\[2 / 2 = 1 \text{ remainder } 0\]
\[0 / 1 = 0 \text{ remainder } 1\]

\[\Rightarrow 58_{10} \rightarrow 111010_2\]

4 Sequence 1: 1, 8, 15, 22, 29, 36, 43, 50, 57, 64, 71, 78, 85, 92, 99, 106, 113, 120, 127, 134, 141, 148, 155, 162, 169, 176, 183, 190, 197, ...

Sequence 2: 1, 10, 19, 28, 37, 46, 55, 64, 73, 82, 91, 100, 109, 118, 127, 136, 145, 154, 163, 172, 181, 190, ...

\[64 + 127 + 190 = 381\]

Geometry

\[\text{area of } \frac{1}{2}(2)(4) = 4\]

entire square area = 64 units²
area of a, b, e, f = 4 units²
area of c = 16 units², area of h, g = 4 units²
entire = a - b - c - e - f - g - h = \[\]
\[64 - 6(4) - 16 = 24 \text{ units}^2 = \text{area of } d\]

There are different ways to solve this.
Think of the image as a $3 \times 4$ rectangle and/or a grid. In a generic $m \times k$ rectangle there are $(m+1)$ vertical grid lines and $(k+1)$ horizontal grid lines. To define any rectangle within the grid, we must choose 2 of each and there are \((m+1)\choose 2\) \times \((k+1)\choose 2\) ways to do that. When you see "choose" think combinations! \[
{\frac{n!}{r!(n-r)!}}
\]

\((m+1)\choose 2 \Rightarrow n = m+1, r = 2\)

\((k+1)\choose 2 \Rightarrow n = k+1, r = 2\)

So, if we have a $3 \times 4$ rectangle
then $m = 3, k = 4$.

\((m+1)\choose 2 \Rightarrow 4\choose 2 \Rightarrow n = 4, r = 2 \Rightarrow 6\)

\((n+1)\choose 2 \Rightarrow 5\choose 2 \Rightarrow n = 5, r = 2 \Rightarrow 10\)

$6 \times 10 = 60$ rectangles

3

each side is $\sqrt{8}$
area of diamond/square $= \left(\sqrt{8}\right)\left(\sqrt{8}\right) = 8$ units$^2$
4. Equation for measuring the angle in degrees between the hour hand and the minute hand:

\[ \left| \frac{1}{2} (60H + M) - 6M \right| \]

\[ H = 8, \ M = 20 \Rightarrow \left| \frac{1}{2} (60(8) + 20) - 6(20) \right| \]

\[ \Rightarrow \left| \frac{1}{2} (500) - 120 \right| = 1250 - 120 \]

\[ = 130^\circ \]

---

**Probability & Statistics**

1. Let \((v+w+x+y+z)/5 = 32\)

\[ \Rightarrow v + w + x + y + z = 160 \]

We're given that \((x+y+z)/3 = 30\)

\[ \Rightarrow x + y + z = 90 \]

\[ \Rightarrow (v+w+x+y+z) - (x+y+z) = (160) - (90) \]

\[ v + w = 70 \]
2. 3 strawberry, 4 orange, 2 lemon $\Rightarrow$ 9 total jellybeans

The probability of Sam drawing a...

- Strawberry is $3/9$, and Mary's chance of drawing a strawberry is $2/8$ $\Rightarrow$ $\frac{3}{9} \times \frac{2}{8} = \frac{1}{12}$ *

- Orange is $4/9$, and Mary's chance of drawing a strawberry is $3/8$ $\Rightarrow$ $\frac{4}{9} \times \frac{3}{8} = \frac{1}{6}$ *

- Lemon is $2/9$, and Mary's chance of drawing a strawberry is $3/8$ $\Rightarrow$ $\frac{2}{9} \times \frac{3}{8} = \frac{1}{12}$ *

* $\frac{1}{12} + \frac{1}{6} + \frac{1}{12} = \frac{1}{3}

3. \begin{array}{cccccccccc}
 n & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
 m & 1 & \frac{2}{2} & 2 & \frac{5}{2} & 3 & \frac{7}{2} & 4 & \frac{9}{2} & 5 \\
 n = m + 4 & x & x & x & x & x & x & x & x & \checkmark
\end{array}

$\Rightarrow$ the minute hand is on the 9

4. \begin{align*}
T T T T & \quad T H H H * \\
T T T H & \quad H T H H * \\
T T H T & \quad H H T H * \\
T H T T & \quad H H H T * \\
H T T T & \quad H H H H * \\
T T H H & \quad H T H T \\
T H H T & \quad T H T H \\
H H H T & \quad H T T H \\
H T T H & \quad H T H T \\
T H T T & \quad \{5/16\}
\end{align*}
1. \(10,000,000 / 9 = 1,111,111\)
\((1,111,111)(9) = 9,999,999\)
\(10,000,000 - 9,999,999 = 1\)
\(1 - 9 = -8\)

2. \(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}\)
\(\frac{1}{2} \times \frac{1}{3} \times \frac{1}{5} = \frac{1}{30}\)
\(\frac{1}{32} < \frac{1}{30}\)

\(\Rightarrow \) half of a third of a 88th is larger

3. \(\frac{1}{2}(x)(60) = 90\)
\(30x = 90\)
\(x = 3\)

\(\text{Area of } \triangle ABD = \frac{1}{2}(x)(5)\)
\(= \frac{1}{2}(3)(5) = \frac{15}{2}\)

\(90 - \frac{15}{2} = \frac{165}{2} \approx 82.5 \text{ cm}^2\)

4. Possible n's according to (a): 36, 43, 50, 57, 64, 71, 78, 85, 92, 99, 106, 113, 120, 127, 134, 141, ...

\(120/13 = 9 \text{ R } 3\)
Mental Math

1. $13 \times 15 = 195$

2. 2 yellow, 6 red $\Rightarrow$ 8 total, $6/8 = \frac{3}{4} = 75\%$

3. $(9 + 6 - 3)/2 = 6$

4. $\frac{2}{3}(36) - \frac{1}{2}(14) = 24 - 7 = 17$

5. 10:55 + 5 min = 11:00
   11:00 + 4 hrs = 3:00
   3:00 + 10 min = 3:10

6. $(7)^2 + (3)^2 = 49 + 9 = 58$

7. $1 / (\frac{1}{3}) = 3$
   $3 \times 12 = 36$

8. Surface area of a cube = $6(s^2)$
   $s = 4 \Rightarrow 6(4^2) = 6(16) = 96$
Individual Answers
1. $1/6$
2. 12 units$^2$
3. 7:03 AM
4. 18 units$^2$
5. 999
6. 55
7. 3
8. 51
9. 3.46
10. 9
11. (15, 75)
12. $x = 12$ or -6

Algebra Answers
1. 126 blorps
2. $49$
3. 9
4. 16 kids

Geometry Answers
1. 64 chairs
2. 8
3. $7\pi$
4. $54\pi$

Probability & Statistics Answers
1. 30
2. 9 students
3. $3/7$
4. 0.15

Potpourri Answers
1. 9
2. 999 perfect squares
3. 300
4. 3
1. Probability of rolling a sum of 10 or higher:
\[ \frac{6}{36} = \frac{1}{6} \]

2. Trapezoid area formula:
\[ \text{area} = \frac{1}{2} (b_1 + b_2)(h) \]

   \[ b_1 = 3 \]
   \[ b_2 = 5 \]
   \[ h = 3 \]

   \[ \frac{1}{2} (3 + 5)(3) = \frac{1}{2} (8)(3) = 12 \text{ units}^2 \]

3. | Alarm Number | Alarm Time  | Time until next alarm |
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<tr>
<th></th>
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<tr>
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</tr>
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<td>7</td>
<td>6:42 AM</td>
<td>6</td>
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<td>5</td>
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<td>4</td>
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<tr>
<td>10</td>
<td>6:57 AM</td>
<td>3</td>
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<tr>
<td>11</td>
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<td>2</td>
</tr>
<tr>
<td>12</td>
<td>7:02 AM</td>
<td>1</td>
</tr>
</tbody>
</table>
   | 13           | 7:03 AM     | 0                     | fog horn
Educated guess and check method:

1. Located the greatest three-digit multiple of 7. I'm doing this because I know that the first three-digit number you will say counting back from 100,000 by 7's will be less than or equal to 7 away from this greatest three-digit multiple of 7. 

   \[(7)(1420) = 994\] ←

2. If 994 was the number we are looking for, then 100,000 - 994 (the range of the numbers you would have theoretically already said) would be a multiple of 7 as well. Is it?

   \[100,000 - 994 = 99006\]

   \[99006 / 7 = 14143.714... \] ← No!

3. So, we're close, but not quite right yet...

   According to step 2, though 994 is not a number that you would say in your list, you will have said 14,143 other numbers before it ... So, what is \[(7)(14143)\]? And could that be a multiple of 7? If so, we can find our number from that:

   \[(7)(14143) = 99001\]

   \[99001 / 7 = 14143 \] ← is a multiple of 7

   \[100,000 - 99001 = 999\]
6  \[ b_1 = 6 \]
\[ b_5 = 34 \]
\[
\begin{array}{c}
6 \\
\hline
- \\
\hline
34 \\
\hline
? \\
\hline
28
\end{array}
\]

Increases 28 units over the course of 4 "jumps"
\[ \Rightarrow \frac{28}{4} = 7 \], each "jump" is up 7.
From 34 to ?, there are 3 "jumps"
\[ \Rightarrow (7)(3) = 21, \ 34 + 21 = 55 = ? = b_8 \]

7 \[ n/7 = x \times 25 \] where \( x \) and \( n \) are integers
Test possible \( n \) and \( 2n \)'s and look for a pattern -

<table>
<thead>
<tr>
<th>( n )</th>
<th>余数</th>
<th>( n/7 )</th>
<th>( 2n )</th>
<th>( 2n/7 )</th>
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<tr>
<td>33</td>
<td>5</td>
<td>66</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \Rightarrow 2n/7 \text{ produces a remainder of } 3 \]

8 \[ (x-2) + x + (x+2) = 2013 \]
\[ 3x = 2013 \]
\[ x = 671 \Rightarrow x-2 = 669 \Rightarrow x+2 = 673 \]

\[ 6 + 6 + 9 + 6 + 7 + 1 + 6 + 7 + 3 = 51 \]
Since we know 60 is not the least common multiple of 60 and n, we know n is not a factor of 60.
Factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60.
But n is a factor of 180.
Factors of 180:
1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 30, 36, 45, 60,
90, 180

So, all possible n's include: 9, 18, 36, 45, 90, 180
Least possible integer value of n = 9

\[ y = 6x - 15 \quad \quad 6x - 15 = x + 60 \]
\[ y = x + 60 \quad \quad 5x = 75 \]
\[ x = 15 \]

\[ y = 15 + 60 = 75 \quad \quad \Rightarrow (x, y) = (15, 75) \]
12 \[ (x)(x) - (x)(6) = 72 \]
\[ x^2 - 6x = 72 \]
\[ x^2 - 6x - 72 = 0 \]
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ x = \frac{6 \pm \sqrt{36 - 4(-72)}}{2} \]
\[ x = \frac{6 \pm 18}{2} = 3 \pm 9 \]
\[ \Rightarrow x = 12 \text{ or } -6 \]

---

**1**

\[ 76 = 15 \]
\[ 35 = 4k \]

\[ 24k = 6(4k) = 6(35) = 6(3(15)) = 6(3\times15) \]
\[ = 6(45) \]
\[ = 120b \]

\[ \Rightarrow \text{Marvin has 120 blops} \]

---

**2**

\[ \begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & 6 \\
7 & 8 & 9 & 3 & 2 & 1
\end{array} \quad \text{-> count} \]

\[ 2(1+2+3+4+5+6) + 7 = 49 \]

---

**3**

\[ 6 \times w \times y = 63 \]
\[ \Rightarrow y + z = 63 \]
\[ x + y = 2 \Rightarrow y + (x+y) = 63 \]
\[ w + x = y \Rightarrow (w+x) + (x+(w+x)) = 63 \]
\[ w + 6 = x \Rightarrow (w + (6+w)) + ((6+w) + (w+(6+w))) = 63 \]
\[ \Rightarrow 5w + 18 = 63 \]
\[ \Rightarrow 5w = 45 \Rightarrow w = 9 \]
Let \( x \) be the number of kids and \( c \) be the number of cats.

\[
2x + 4c = 128 \quad (\text{legs})
\]
\[
x + c = 40 \quad (\text{heads}) \Rightarrow c = 40 - x
\]

\[
2x + 4(40 - x) = 128
\]
\[
2x + 160 - 4x = 128
\]
\[
-2x = -32
\]
\[
x = 16 \quad \Rightarrow \text{there are 16 kids}
\]

**Geometry**

1. Maximize the sides with 3 seats!

   ![Diagram](image)

   \[
   \text{Seats} = 20(3) + 2(2) = 64
   \]

2. Just count the boxes:

   - Perimeter of figure on the left (square) = 20
   - Perimeter of figure on the right (diamond) = 28

   \[
   28 - 20 = 8
   \]

3. The standard arc is characterized by only 3 letters: That’s why it wasn’t explicitly mentioned.

   arc through \( C, E, D \) is the section of the circumference that connects the points \( C, B, E, D \) in that order = \( x \)

   circumference = \( 2(6)\pi = 12\pi \)

   \[
   30^\circ + 150^\circ + 30^\circ = 210^\circ \quad \text{angle associated with} \ x
   \]

   \[
   \frac{210^\circ}{360^\circ} = \frac{x}{12} \Rightarrow x = \frac{7}{12} \text{ of the circumference}
   \]

   \[
   x = \frac{7}{12} (12\pi) = \frac{7\pi}{3}
   \]
Area of a semicircle = \( \frac{\pi r^2}{2} \)

Area of larger semicircle:
\[ \frac{\pi (24)^2}{2} = 288\pi \]

Area of smaller semicircle:
\[ \frac{\pi (12)^2}{2} = 72\pi \]

Area in the larger semicircle but not in the smaller semicircle:
\[ 288\pi - 72\pi = 216\pi \]

216\pi divided into 4 equal sections = 54\pi

Probability & Statistics

1. Options for first place: 6
Options for second place after first place has been awarded: 5
\[ (6)(5) = 30 \]

2. 12 outside of either circle

50 - 12 = 38

\[ (22-x) + x + (25-x) = 38 \]

\[-x = -9 \]

\[ x = 9 \]

9 students like both!
### Table 3

<table>
<thead>
<tr>
<th>1</th>
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<th>4</th>
<th>5</th>
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<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>

Most common outcomes: 15 most common outcomes

\[ \frac{15}{35} = \frac{3}{7} \]

### Diagrams

1. Squares and circles with formulas:
   - Area of square: \( s^2 \)
   - Area of circle: \( \pi \left( \frac{s^2}{4} \right) \)
   - Area of target: \( s^2 \)

2. Ratios:
   - \( \frac{\text{Area of square}}{\text{Area of target}} = \frac{s^2}{\pi \left( \frac{s^2}{4} \right)} \)
   - \( \frac{\text{Area of circle}}{\text{Area of target}} = \frac{\pi \left( \frac{s^2}{4} \right)}{s^2} \)

3. Calculations:
   - \( \frac{\pi}{4} - \frac{2}{\pi} = 0.15 \)
Potpourri

<table>
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<th>Combo #</th>
<th>basket a</th>
<th>basket b</th>
<th>basket c</th>
<th>basket d</th>
<th>basket e</th>
<th>basket f</th>
<th>basket g</th>
<th>basket h</th>
<th>Total</th>
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<td>3</td>
<td>3</td>
<td>24</td>
</tr>
</tbody>
</table>

a - h represent the numbers 1 - 8 in no particular order. Order does not matter for this problem.

2

Is 1,000,000 a perfect square? Yes! 
\[ \sqrt{1,000,000} = 1,000 \Rightarrow \text{there are 1,000 perfect squares between 0 and 1,000,000 inclusive.} \]
But we want all the perfect squares strictly less than 1,000,000 (non-inclusive), so...

\[ 1,000 - 1 = 999 \]
\[ \Rightarrow \text{There are 999 perfect squares less than 1,000,000} \]

3

Well, how many times will one write an 8 for the numbers 0 - 100? 20 times (8, 18, 28, 38, 48, 58, 68, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 98).

There are 10 intervals of 100 from 1 - 1,000, and one of those intervals has 100 additional 8's in it (800 - 899).

\[ 10 \times 20 + 100 = \boxed{300} \text{ 8's} \]
\[ a^2 - b^2 = 6 \]
\[ a - b = 2 \quad \Rightarrow \quad a = 2 + b \]
\[ (2 + b)^2 - b^2 = 6 \]
\[ b^2 + 4b + 4 - b^2 = 6 \]
\[ 4b + 4 = 6 \]
\[ 4b = 2 \]
\[ b = \frac{1}{2} \]
\[ a - \frac{1}{2} b = 2 \]
\[ a = \frac{5}{2} \]
\[ b = \frac{1}{2}, \quad a = \frac{5}{2} \quad \Rightarrow \quad a + b = \frac{5}{2} + \frac{1}{2} = \frac{6}{2} = 3 \]

**Mental Math**

1. \[ \text{Area} = \omega L \quad \text{Perimeter} = 2 \omega + 2L \]
   \[ 64 = (4x)(x) \quad = 2(4x) + 2(x) \]
   \[ 64 = 4x^2 \quad \Rightarrow \quad 2(16) + 2(4) \]
   \[ 16 = x^2 \quad = 32 + 8 \]
   \[ 4 = x \quad = 40 \]

2. \[ 6 + 2 = 8 \]

3. \[ 23 - 9 = 14 \quad (14)^2 = 196 \]

4. \[ 10a \Rightarrow 1(a^1) + 0(a^0) = 1(a^1) = a^1 \]

5. \[ \text{Feb 15th: } \frac{x}{3}, \quad \text{Feb 16th: } \frac{1}{3} \left( \frac{x}{3} \right) = \frac{x}{9}, \quad \text{Feb 17th: } G \]
   \[ x - \frac{x}{3} - \frac{x}{9} = 6 \quad \Rightarrow \quad \frac{x}{3} = 6 \quad \Rightarrow \quad x = 24 \]

6. \[ 95/2.5 = \boxed{38} \text{ MPH} \]

7. \[ (42)(19) = \boxed{798} \]

8. \[ (90)(0.9) = \boxed{81} \]
Whatcom County Math Championship
2014 Mock Test Answers

**Individual Answers**
1. 64%
2. 9
3. 2/5
4. 720720
5. 6 sides
6. 50
7. 10
8. 75
9. 60
10. 35
11. 5
12. 38

**Algebra Answers**
1. 64
2. 95
3. 7
4. (12,15)

**Geometry Answers**
1. 1728 cubic inches
2. $63\pi$
3. 2.5
4. 20

**Probability & Statistics Answers**
1. 5/12
2. 3/8
3. 19/10000
4. 20/61

**Mental Math Answers**
1. 12
2. 49
3. 24
4. 85
5. 9
6. 11
7. 26
8. 42

**Potpourri Answers**
1. –12
2. 3
3. 1260
4. 12
Individual

2. Free throws @ 80%  
   \( P(n \text{ FT's in a row}) = (FT\%)^n \)
   \( (0.8)^2 \)
   \(.8 \times .8 = .64 = 64\% \)

3. \( \triangle \)
   \( S^2 + S^2 = 18^2 \)
   \( 2S^2 = 18 \)
   \( S^2 = 9 \)
   \( S = 3 \)

3. \( \frac{49}{12} = \frac{12}{30} = \frac{2}{5} \)

4. 12 hrs = 720 min  
   1001 nights = 720,720

5. \( \\)
   \( 120 \cdot 6 = 720 \)
   \( 240 \cdot G = 1440 \)
   \( \frac{1440}{720} = 2 \)

6. \( \frac{100}{2} + \frac{96}{2} - \frac{94}{2} + \ldots + \frac{8}{2} - \frac{6}{2} + \frac{4}{2} - \frac{2}{2} \)
   \( \text{Answer} = 6 \text{ sides} \)

50 pairs  
2 each = 50
7. $10 : 101, 103, 107, 109, 113, 127, 131, 137, 139, 149$

8. \[ \frac{27}{0.36} = 75 \] \[ 75 \cdot 0.36 = 27 \]

9. \[ 13^2 + x^2 = 13^2 \] \[ 144 + x^2 = 169 \] \[ x^2 = 25 \] \[ x = 5 \]

10. \[ x + y = 58 \] \[ 58 - 12 = 46 \] \[ 23 + 35 = 58 \] \[ x - y = 12 \] \[ 46 / 2 = 23 \] \[ 35 - 23 = 12 \] \[ 23 + 12 = 35 \]

11. 32 teams
   16 games
   16 teams
   8 games
   8 teams
   4 games
   4 teams
   2 games
   2 teams
   1 game

12. 38: 4, 9, 16, 25, 36, 49, 8, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625, 676, 729, 784, 841, 900, 961 (end of 38)

27, 125, 216, 343, 512, 729, 1000 (end of 38)
1. \[ \begin{align*}
\text{Step 1:} & \quad 7 > 3 \\
\text{Step 2:} & \quad 10 > 3 \\
\text{Step 3:} & \quad 13 > 3 \\
\text{Step 4:} & \quad 16 > 3 \\
\text{Step 5:} & \quad 19 > 3 \\
\text{Step 20:} & \quad 64 \\
\end{align*} \]

\[ 20 - 1 = 19 \]

\[ 3 \cdot 19 = 57 + \text{Step 2} = 64 \]

2. \[ \begin{align*}
xy + yx &= 154 \\
95 + 59 &= 154 \\
86 + 68 &= 77 + 77 \\
68 + 86 &= 59 + 95 \\
\end{align*} \]

3. \[ 9 + 2 + 0 + 1 + 4 = 16 \]

\[ 1 + 6 = 7 \]

4. \[ (4, 21) \text{ and } (28, 3) \]

\[ y = \frac{-3}{4}x + 24 \]

\[ x_2 - x_1 \]

\[ \frac{3 - 21}{28 - 4} = \frac{-18}{24} = -\frac{3}{4} \]

\[ y = \frac{-3}{4}x + b \]

\[ 21 = \frac{-3}{4}(4) + b \rightarrow b = 24 \]

\[(16, 27) \text{ slope of } 3 \]

\[ y = 3x + b \]

\[ 27 = 3(16) + b \rightarrow b = -21 \]

\[ y = 3x - 21 \]

\[ y = 3(12) - 21 \]

\[ y = 15 \]

\[ (12, 15) \]

\[ 180 = 15x \]

\[ x = 12 \]
1. Geometry

\[ xy = 108 \]
\[ x = 9 \]
\[ \frac{xy}{2} = 192 \]
\[ y = 12 \]
\[ 2x = 144 \]
\[ x = 72 \]
\[ y^2 = 1728 \]
\[ 2x = ? \]
\[ \frac{xy}{2x} = \frac{108 \cdot 144}{9} = 15552 \]
\[ 1728 \]

2. Area of a circle

\[ 81 \pi \text{ cm}^2 = \pi r^2 \]
\[ d = 18 + 6 = 24 \]
\[ 81 = r^2 \]
\[ r = 9 \]
\[ r^2 = 144 \]
\[ d = 18 \]
\[ \pi r^2 = 144 \pi \]
\[ 144\pi - 81\pi = 63\pi \]

3. Time and speed

\[ \frac{360°}{12} = 30° \]
\[ \left| \frac{1}{2} (60H + M) - GM \right| = 2.5 \]
\[ \left| \frac{1}{2} (60(10) + 55) - 6(55) \right| = 2.5 \]

4. Geometry

\[ (1x)^2 + (2x)^2 = (5x)^2 \]
\[ x^2 + 4x^2 = 25x^2 \]
\[ \sqrt{5x} \cdot \sqrt{5x} \]
\[ 5x^2 = 100 \]
\[ x = \sqrt{20} \]
\[ \frac{1}{2} (20)(20) \]
\[ x = 100 \]
\[ 20 \cdot 4 = 80 \]
1. **Prob & Stats**

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<tr>
<td>6 18 18 24 30 36</td>
<td>6 18 18 24 30 36</td>
</tr>
</tbody>
</table>

\[
\frac{15}{36} = \frac{5}{12}
\]

2. \(2 \cdot 2 \cdot 2 \cdot 2 = 16\)

3. \(\frac{19}{10000} = 0.0019\)

4. Numbers whose digits sum to be a multiple of 3 are divisible by 3.

Numbers from 20 to 80 who are divisible by 3:

21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51,
54, 57, 60, 63, 66, 69, 72, 75, 78

Number of numbers for 20 to 80 inclusive:

\((80 - 20) + 1 = 61\)
1. \[
\frac{1000000}{18} - \frac{999999}{1} = 12
\]

2. \[a^2 + b^2 = c^2\]
\[a^2 + b^2 = c^2\]
\[x = 3\]

3. **ARRANGE**

7 letters → 7! ways to arrange them

A 2
R 2
N
G
E

2520 → 1260

4. \[23x - 314 = 30\]

19x = 204 = 24

23
8° 8'

2 \cdot 8' = 16
3 \cdot 8° = 3

19

4 \cdot 8° = 4
1 \cdot 8' = 8
3 \cdot 8° = 30

24

8° 8'

3 \cdot 8° = 192

204
Mental Math

2. \[ A = \frac{36\pi}{2} \quad \frac{36\pi}{\pi} = 36 \quad r = 6 \]

3. \[ 1 + 3 + 5 + 7 + 9 + 11 + 13 = 49 \]

4. \[ 2(18) = \frac{36}{3} = 12 \quad 36 - 12 = 24 \]

5. \[ 6^2 = 36 \quad 121 - 36 = 85 \]

6. \[ 11^2 = 121 \]

5. \[ 1:4, 1:3, 1:5 \]

6. \[ 2:4, 2:5, 2:6, 3:5, 3:6, 4:6 \]

7. \[ x + y = 43 \]
   \[ x = 17 \]
   \[ y = 26 \]

8. \[ 60 \times 0.70 = 42 \]
Individual Answers
1. $5.26
2. 10 combinations
3. 72°
4. 16 vertices
5. 4/9
6. 9
7. 3/16
8. 40
9. 432
10. 9
11. 3 inches
12. –14

Algebra Answers
1. 2015
2. 27.47
3. 9
4. 4

Geometry Answers
1. (10, 11)
2. 150°
3. 14
4. –6

Probability & Statistics Answers
1. 1/3
2. 196th
3. 1/6
4. 0.41

Potpourri Answers
1. 36 times
2. 181
3. 81
4. 2 bagels
Individual

1. 3 books @ $3.95 each cost $11.85.
   60% of $11.85 is $7.11
   $11.85 - $7.11 = $4.74 is the new cost for the books.
   $10 - $4.74 = $5.26 is his change

2. Method 1: List the possible combinations
   chocolate (ch), vanilla (v), blueberry (bl)
   cinnamon (c), and butterscotch (b) are the flavors.
   Possible combinations:
   (ch, v), (ch, bl), (ch, c), (ch, b), (v, bl),
   (v, c), (v, b), (bl, c), (bl, b), (c, b)
   → 10 combinations

Method 2: Use the combination formula
Formula: \( \frac{n!}{r!(n-r)!} \)
   \( n = \# \text{ of possible flavors} \)
   \( r = \# \text{ flavors Alexia chooses} \)

\[
\frac{5!}{2!(5-2)!} = \frac{5!}{2!3!} = \frac{120}{(2)(6)} = 10 \text{ combinations}
\]
Ratio of the angles of the triangle: \(1:2:2\)

Let the smallest angle be \(x\) degree. That implies the other two angles are of degree \(2x\) according to the ratio. We know the interior angles of a triangle sum to \(180^\circ\), so...

\[
x + 2x + 2x = 180^\circ \\
5x = 180^\circ \\
x = 36^\circ \quad \text{The bigger angle is } \; 2x = 72^\circ
\]

4 This is what a hypercube looks like:

![Hypercube Diagram]

Basically it's a cube within a cube, each cube has 8 vertices.

5 \(3.375 = 3\frac{3}{8} = \frac{27}{8}\)

- Reciprocal: \(\frac{8}{27}\)
- Cube root of reciprocal: \(\sqrt[3]{\frac{8}{27}} = \frac{2}{3}\)
- Square of cube root of reciprocal: \((\sqrt[3]{\frac{8}{27}})^2 = (\frac{2}{3})^2 = \frac{4}{9}\)
6 \( (3)(x) = 6 \quad 6 \div 5 = 1 \ R 1 \quad \Rightarrow \text{2 is the inverse of 3 mod 5} \)

So we need to find \( x \), where...

\( (3)(x) = y \quad y \div 13 = z \ R 1 \)

Let's work backwards! We know \( y \) is one more than a multiple of 13 and it is a multiple of 3.

Multiples of 13: \(\{13, 26, 39, 52, \ldots\} \)

This implies that the set of numbers that are one more than 13 are: \(\{14, 27, 40, 53, \ldots\} \)

Multiples of 3: \(\{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, \ldots\} \)

\( \Rightarrow y \) must be 27.

\( (3)(x) = y \quad \Rightarrow \quad (3)(x) = 27 \quad \Rightarrow \quad x = 9 \)

7

25\% = 0.25 = \( \frac{25}{100} = \frac{1}{4} \)

\[ \frac{3}{4} \times \frac{1}{4} = \frac{3}{16} \]

8

Geometric sequence: 8, \( m \), 72 \( \Rightarrow 8x = m \) & \( 8x^2 = 72 \)

\( 8x^2 = 72 \quad \Rightarrow \quad x^2 = 9 \quad \Rightarrow \quad x = 3 \)

\( 8x = m \quad \Rightarrow \quad 8(3) = m \quad \Rightarrow \quad m = 24 \)

\[ \Rightarrow \text{means "implies"} \]

Arithmetic sequence: 8, \( m \), ? \( \Rightarrow 8, 24, ? \)

\( 24 - 8 = 16 \quad \Rightarrow \quad ? - 16 = 24 \quad \Rightarrow \quad ? = 40 \)
Because the area of one of the small squares is 36, we know its side lengths are 6. This implies that the one large square has side lengths of 24. Furthermore, the dimensions of the rectangle are 18 x 24.

\[ \text{Area of the rectangle} = (18)(24) = 432 \]

We need to know the shape of the given parabola. So, what would it look like if \( m \) (the y-intercept) was zero? 

From the graph, you can see that the minimum exists at \((3, -9)\). If we were to shift the parabola up 9 units that minimum would exist on the x-axis. Thus, the parabola would have one x-intercept. In order for it to have no x-intercepts, we would have to move it up more than 9. When we "move up" a graph, we are essentially increasing the y-intercept \( m \). Thus, we can conclude from what was previously stated that if \( m > 9 \), the parabola would have no x-intercepts. Thus, the smallest value of \( m \) is 9.
March 14th → 30th = 16 days
Triple 2 feet = 6 feet
So, in 16 days, Rapunzel’s hair grew 4 feet (48 inches).

\[
\frac{48 \text{ inches}}{16 \text{ days}} = \frac{3 \text{ inches}}{1 \text{ day}} \Rightarrow 3 \text{ inches per day}
\]

\[y = x^2 - 10x - 24\]

Factor: \[y = x^2 - 10x - 24 = (x - 12)(x + 2)\]

\[\Rightarrow \text{x-intercepts} = 12, -2\]

The y-intercept can be found by solving \[y = x^2 - 10x - 24\] when \[x = 0\].

\[\Rightarrow y \text{-intercept} = -24\]

\[-24 + 12 - 2 = -14\]

Algebra

\[\begin{align*}
\text{Sum of evens: } & 2 + 4 + 6 + 8 + ... \\
\text{Sum of odds: } & 1 + 3 + 5 + 7 + ... \\
\text{Difference: } & 1 + 1 + 1 + 1 + ... \\
\end{align*}\]

Since we are looking at the first 2015 even & odd counting numbers, this implies that the difference between the two sums is 2015.
2. \(4, \_\_\_, \_\_\_, 72\) is the geometric sequence
\[
\begin{align*}
4x & \quad 4x^2 & \quad 4x^3 \\
\Rightarrow 4x^3 & = 72 & \Rightarrow x^3 & = 18 \\
\Rightarrow x & = (18)^{\frac{1}{3}} \approx 2.62074
\end{align*}
\]
Third term \(= 4x^2 = 4(1.5)^2 \approx 27.47\)

3. \(8^x = (2^3)^x = (3^3) = 27 \Rightarrow \sqrt[3]{(2^3)^x} = \sqrt[3]{(3^3)}\)
\[ \Rightarrow 2^x = 3 \]
\[ 4^x = (2^x)(2^x) = (\_\_\_)(3) = 9 \]

4. \(24, 36, 54, ...\) is the geometric sequence
\[
\begin{align*}
24x & \quad 24x^2 \\
\Rightarrow 24x & = 36 & \Rightarrow x & = 1.5
\end{align*}
\]
If the second term is equivalent to \(24x\) and the third term is equivalent to \(24x^2\) we can conclude that the 9th term will be equivalent to \(24x^8\).
\[
24x^8 = 24(1.5)^8 = 615.09375 = 615 \frac{3}{52}
\]
\[
615 \frac{3}{52} = 19683 = \frac{3^9}{2^5} \Rightarrow n=9 \quad m=5
\]
\[ n-m = 9-5 = 4 \]
Geometry

1. Congruent circles have congruent radii lengths. Furthermore, in this particular situation, we know that the radii that connect the centers and the point of tangency have the same slope. If they didn't the circles wouldn't be tangent or wouldn't be congruent because both radii have to be perpendicular to a single line that is tangent to both circles at the point (8,14).

Using that knowledge and the given coordinates we can conclude that the center of the second circle is at (10,11) as depicted above.

2. \[90^\circ + 60^\circ + 60^\circ + x^\circ = 360^\circ\]
   \[210^\circ + x^\circ = 360^\circ\]
   \[x^\circ = 150^\circ\]

3. \[y = 4 - 1x - 21\]

   \[8 + 4 + 2 = 14\]
We know that a rectangle has two sets of congruent parallel sides. After sketching out the three given points, we can thus conclude that the side from (-2, -6) to \((m, n)\) must be parallel to the side connecting \((4, 6)\) and \((8, 4)\).

The slope of the side connecting \((4, 6)\) and \((8, 4)\) is \(-\frac{2}{4}\). Thus, \((m, n)\) is \((-2 + 4, -6 - 2) = (2, -8)\).

\[
m + n = 2 - 8 = -6
\]

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**Probability & Statistics**

1. Positive factors of 72:
   \[
   \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72\}
   \]

   Positive factors of 72 that are less than 5:
   \[
   \{1, 2, 3, 4\}
   \]

   \[
   \frac{4}{12} = \frac{1}{3}
   \]
Available letters: A, B, E, I, L, N.

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<th>Letter</th>
<th>Possible Combinations</th>
<th>Words</th>
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<tr>
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<td>4 \times 3 \times 2 \times 1</td>
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<tr>
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<td>N</td>
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196 words

BLAINE is 196th

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3.

The shaded area of the rectangle is the area that includes all the points \((x, y)\) where \(x \leq y\).

Area of the rectangle: 
\((6)(2) = 12\)

Area of the shaded area: 
\(\frac{1}{2}(2)(2) = 2\)
Area of shaded square: \( \frac{1}{4} \)
Area of one shaded triangle: \( \frac{1}{2} \left( \frac{1}{\sqrt{2}} \right) \left( \frac{1}{\sqrt{2}} \right) = \frac{1}{4} \)
Total shaded area: \( 1 + 4 \left( \frac{1}{4} \right) = 2 \)
Area of one unshaded rectangle: \( \left( \frac{1}{\sqrt{2}} \right) \left( 1 \right) = \left( \frac{1}{\sqrt{2}} \right) \)
Total area of the octagon: \( 2 + 4 \left( \frac{1}{\sqrt{2}} \right) = 2 + 2\sqrt{2} \)

\[
\Rightarrow \frac{a}{a + 2\sqrt{2}} = \frac{1}{1 + \sqrt{2}} \approx 0.41
\]

Potpourri:

1. Valid times:
   1:35, 1:37, 1:39, 1:53, 1:57, 1:59,
   3:15, 3:17, 3:19, 3:51, 3:57, 3:59,
   5:13, 5:17, 5:19, 5:31, 5:37, 5:39,
   7:13, 7:15, 7:19, 7:31, 7:35, 7:39,
   7:51, 7:53, 7:59, 9:13, 9:15, 9:17,

\( \Rightarrow 36 \) times
Method 1: Pattern recognition

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<th>Stars Added</th>
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<td>9</td>
<td>145</td>
<td>17 + 19</td>
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There's a pattern in the # of stars you add.

Method 2: Create a formula
Let \( n \) = the figure number
Let \( S_n \) = the star count for figure \( n \)
Let \( p_n \) = the \( n^{th} \) odd number

\[
S_n = S_{n-1} + p_{n-1} + p_n
\]

Use this formula to then fill out the above table.
3 \( (11111111)^2 = 12345678987654321 \)
\[1+2+3+4+5+6+7+8+9+8+7+6+5+4+3+2+1 = 81\]

4 Let \( m \) = number of muffins Grace bought
Let \( b \) = number of bagels Grace bought

\[
m(50) + b(75) = c
\]
\[m + b = 5\]

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\[\Rightarrow \text{Grace bought 2 bagels}\]

Mental Math

1 dice two

dice one

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Unique outcomes have been circled once.

\[\Rightarrow 11 \text{ outcomes}\]
2. 110 hours = 4 days and 14 hours
   14 hours after noon = \(2\) AM

3. Square with area \(36\) has side lengths of \(6\).
   Circles of radius \(2\) have a diameter of \(4\).
   \(\frac{6}{4} = 3\) ⇒ you can fit 3 circles across
   and 3 circles down inside the square
   \(3 \times 3 = 9\) total circles

4. Six-dozen = 72
   \(72 + \frac{1}{2} = 72.5\)

5. To get from midnight to 2:45 AM the minute
   hand completes \(2\frac{3}{4}\) rotations around the clock.
   \(2\frac{3}{4} = 2.75\) \(2.75 \times 360 = 990°\)

6. \(2015 - 5102 = -3087\)

7. 5 diagonals

8. You can arrange 6 equilateral triangles
   in a regular hexagon. A line of symmetry cuts the hexagon in half. So, now you only have 3
   equilateral triangles arranged as so:
   
   \[\text{perimeter} = (6)(5) = 30\]